

# Review of Unsteady Vortex Flows over Slender Delta Wings

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## Nomenclature

AR	=	aspect ratio; amplitude ratio
$B$	=	probability density function of velocity
$c$	=	root chord length
$f$	=	frequency
$k$	=	reduced frequency; axial wave number
$n$	=	wave number in angular direction
$P$	=	probability
$p$	=	pressure fluctuation
$Re$	=	Reynolds number based on chord length
$S$	=	spectral density
$s$	=	local semispan
$T$	=	period
$t$	=	time
$U_\infty$	=	freestream velocity
$u$	=	axial velocity
$v$	=	swirl velocity
$x$	=	streamwise distance
$x_{bd}$	=	breakdown location
$y$	=	spanwise distance
$z$	=	vertical distance above wing surface
$\alpha$	=	angle of attack
$\Gamma$	=	circulation
$\delta$	=	flap angle
$\Lambda$	=	sweep angle
$\nu$	=	kinematic viscosity
$\tau$	=	time constant
$\Phi$	=	fin angle
$\omega$	=	vorticity; radial frequency

## I. Introduction

**H**IGHLY swept wings, often referred to as delta wings due to their triangular planform, are used in a variety of aerospace vehicles. At high angles of attack, delta wings can generate higher lift than rectangular wings, with better aircraft stability and control characteristics. The development of highly maneuverable fighter aircraft and missiles has raised interest in the study of delta wings.

The flow over a delta wing at high angles of attack is dominated by two large, counter-rotating leading-edge vortices that are formed by the roll-up of vortex sheets, as shown in Fig. 1. The flow separates from the leading edge of the wing to form a curved free shear

layer above the suction side of the wing, which rolls up into a core. The time-averaged axial velocity is roughly axisymmetric, and its maximum can be as large as four or five times the freestream velocity. These large axial velocities are due to very low pressures in the vortex core that generate additional suction and lift force on the delta wings.

At a sufficiently high angle of attack, the vortices undergo a sudden expansion known as vortex breakdown, which was first observed by Werle<sup>1</sup> in 1954 in a water-tunnel facility. Vortex breakdown has adverse effects on the time-averaged performance. For example, the magnitude of the lift and pitching moment decreases after vortex breakdown for slender wings. A great deal of effort has been focused on the study of these vortices, vortex breakdown phenomenon, and aerodynamics of delta wings, as summarized in several review articles.<sup>2–4</sup>

There has been less emphasis on the unsteady aspects of these flows, which have an impact on aircraft stability and control and wing/fin buffeting. For example, vortex breakdown may cause large structural vibrations and severe fatigue damage of fins. The dynamic response of leading-edge vortices and breakdown is important for the flight of modern fighter aircraft. It is crucial to understand these unsteady vortex flows to ensure successful, highly maneuverable aircraft. Unsteady flow structure and aspects of the vortex breakdown phenomenon were reviewed by Rockwell<sup>5</sup> and Visbal<sup>6</sup>. Unsteady aerodynamic loading of delta wings was reviewed by Ashley et al.<sup>7</sup> The objective of this paper is to review a wide range of unsteady phenomena of leading-edge vortices over stationary and dynamic slender delta wings.

## II. Shear Layer Instabilities

The separated shear layers shown in Fig. 1 roll up periodically into discrete vortical substructures, as visualized by Gad-el-Hak and Blackwelder,<sup>8</sup> at low Reynolds number flow over a delta wing. This phenomenon was attributed to a Kelvin–Helmholtz type instability of the shear layer. The unsteady Kelvin–Helmholtz (K–H) instability has been observed via flow visualization,<sup>8</sup> hot-wire velocity measurements,<sup>9</sup> particle image velocimetry measurements,<sup>10,11</sup> and numerical simulations.<sup>12</sup> Riley and Lowson<sup>13</sup> suggested that the appearance of the unsteady K–H instability is due to extraneous inputs peculiar to the particular wind/water tunnel and that this unsteady instability is not a generic part of the flow. They reported that the appearance of the unsteady instability was dependent on



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a certain range of tunnel velocities. Also, Lowson<sup>14</sup> found that the shear layer was forced by vibrational inputs of the tunnel motor cooling fan running at a constant speed of 50 Hz. Note that both Gad-el-Hak and Blackwelder<sup>8</sup> and Lowson<sup>15</sup> in another publication reported that the frequency of the unsteady instability varies with the freestream velocity. It is well known that disturbances in individual facilities may cause wide scatter of dominant frequencies for two-dimensional shear layers<sup>16,17</sup> because the shear layer is unstable to a wide range of frequencies. However, in the absence of any discrete disturbances, the instability will develop at the most unstable frequency. This is the case in the numerical simulations<sup>12</sup> in which no deliberate forcing of the shear layer is applied. In both experiments<sup>10</sup> and computations,<sup>12</sup> the observed frequencies were found to agree with the predictions from the linear stability analysis of the crossflow shear layer. Therefore, the unsteady K-H instability over delta wings is a generic part of the flow as much as it is for two-dimensional shear layers. Another important feature of the unsteady K-H instability is that it exists in both laminar and turbulent mixing layers.<sup>17</sup> Likewise, these vortical structures were detected over delta wings by the use of hot-wire measurements<sup>9</sup> at Reynolds numbers much higher than those corresponding to flow visualization experiments. It was suggested<sup>12</sup> that the shear layer

unsteadiness and roll up are closely linked to the vortex/surface interaction and boundary-layer separation. The relation of the onset of shear layer instabilities to the unsteady boundary-layer separation needs to be investigated for higher Reynolds numbers.

Several researchers<sup>13,18,19</sup> revealed the existence of stationary, small-scale vortices around the primary vortex. Such steady structures can persist even after vortex breakdown, as shown in Fig. 2 (from Mitchell and Molton<sup>18</sup>). The origin of these steady structures is not well-understood and has been the subject of a variety of hypotheses. The spatially fixed substructures were measured by velocity probes at fixed locations and were identified as a result of time averaging the flow. Their relation to the temporal substructures was recently demonstrated by direct numerical simulations.<sup>20</sup> Instantaneous flow images (Fig. 3a) show the unsteady substructures and the transition from laminar to turbulent flow with increasing Reynolds number. More significantly, the time-averaged vorticity isosurfaces show the steady vortical substructures (Fig. 3b). These results indicate that steady and unsteady substructures are not necessarily two separate phenomena.

### III. Vortex Wandering

It was shown by Menke and Gursul<sup>21</sup> that large-amplitude velocity fluctuations occur upstream of breakdown and also in the absence of breakdown over delta wings. The variation of rms swirl velocity is shown in Fig. 4. The rms swirl velocity is large within the subcore and has a maximum at the axis of the time-averaged vortex. The maximum rms swirl velocity can exceed the freestream velocity, depending on the angle of attack. Other investigators also observed large velocity fluctuations in vortex cores over delta wings,<sup>22,23</sup> an aircraft model,<sup>24</sup> and an ogive cylinder<sup>25</sup> over a wide range of Reynolds numbers. Large velocity fluctuations in the vortex cores are common regardless of geometry and Reynolds number. Note that the amplitude of the velocity fluctuations depends on the time-averaged velocity, which is a function of particular geometry and angle of attack. It was shown that maximum rms swirl velocity is roughly one-half of maximum time-averaged swirl velocity.

It was suggested that these large-amplitude, broadband random velocity fluctuations are due to random displacements of the vortex core. This wandering of the vortex core was also observed in tip vortices trailing from rectangular wings.<sup>26-28</sup> A useful statistical quantity that was employed by Menke and Gursul<sup>21</sup> to study these velocity fluctuations is the probability density function, which is a measure of the relative amount of time that the velocity spends at various levels. It was shown that the probability density function of swirl velocity is very close to the Gaussian near the vortex axis,

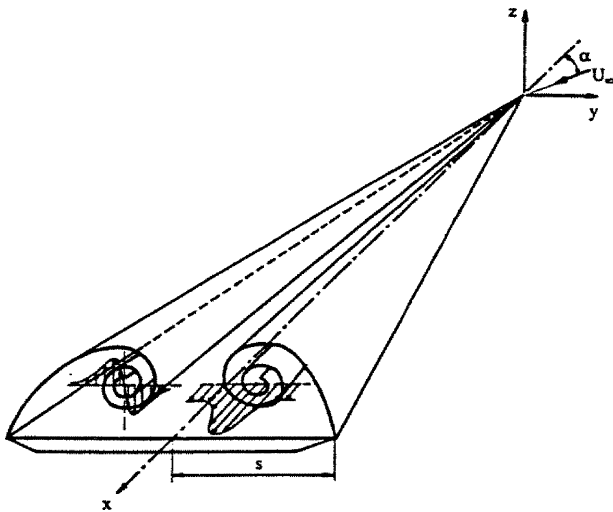


Fig. 1 Schematic of shear layer and leading-edge vortices over a delta wing.

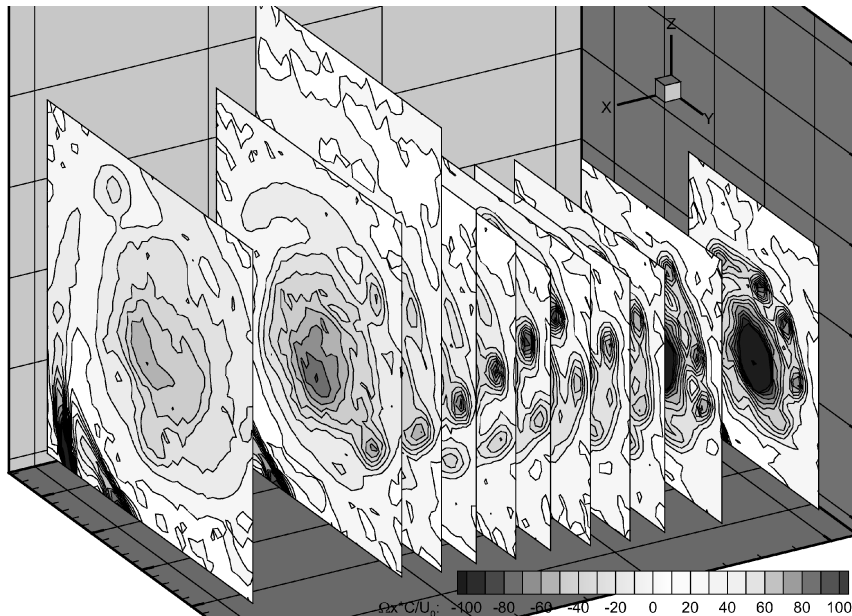


Fig. 2 Vortical substructures around the primary vortex over a delta wing.<sup>18</sup>

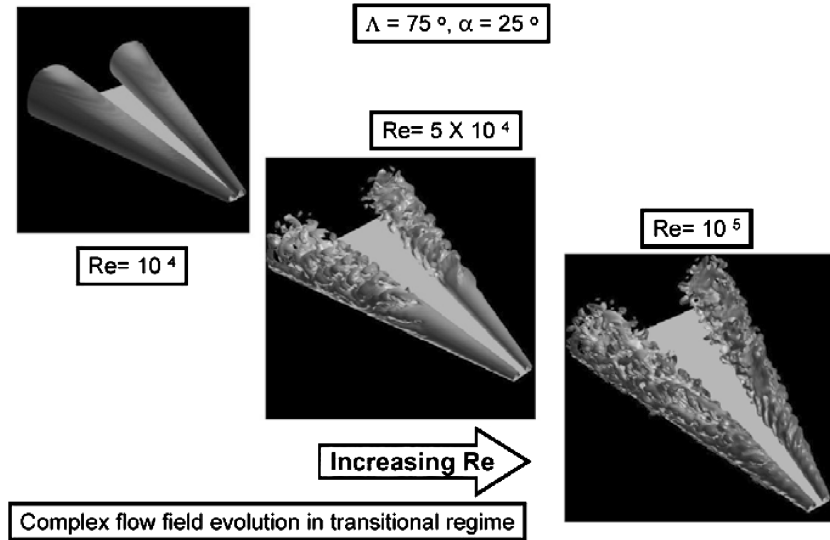


Fig. 3a Instantaneous flow showing the transition process with increasing Reynolds number.

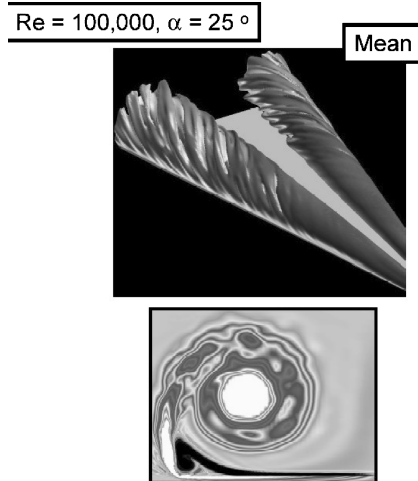


Fig. 3b Time-averaged flow showing mean vortical substructures.<sup>20</sup>

but becomes similar to a lognormal distribution away from the axis (Fig. 5a). Menke and Gursul<sup>21</sup> proposed a simple model of the flow in which the axis of the Q vortex has random displacements around a mean location. This simple model produced all of the essential features of the measured probability density functions as seen from the comparison of laboratory and model flows (Fig. 5b).

Several possibilities for the origin of vortex wandering were suggested. For example, it was suggested by Baker et al.<sup>26</sup> and Devenport et al.<sup>27</sup> that vortex wandering in tip vortices is due to freestream turbulence. However, for leading-edge vortices, Menke and Gursul<sup>21</sup> showed that vortex core displacements are much larger than those caused by freestream turbulence. Several possibilities, including the K-H instability in the shear layer and unsteady turbulent flow in the wake of the wing, were discussed as potential sources of vortex wandering over delta wings. It is known that shear layer vortices due to the K-H instability exist in the separated shear layer. However, as can be seen from Fig. 4, the swirl velocity fluctuations rapidly decrease with the radial distance from the centerline of the time-averaged vortex, and the fluctuations in the shear layer are not as energetic. Whether these weak fluctuations in the shear layer can induce very large fluctuations in the vortex core has not been clear. This issue was later clarified by comparison of the velocity fluctuations in the absence and presence of the K-H instability.

As shown by Lowson,<sup>14</sup> at low Reynolds numbers the flow is laminar and the separated shear layer is steady (Fig. 6a). As the

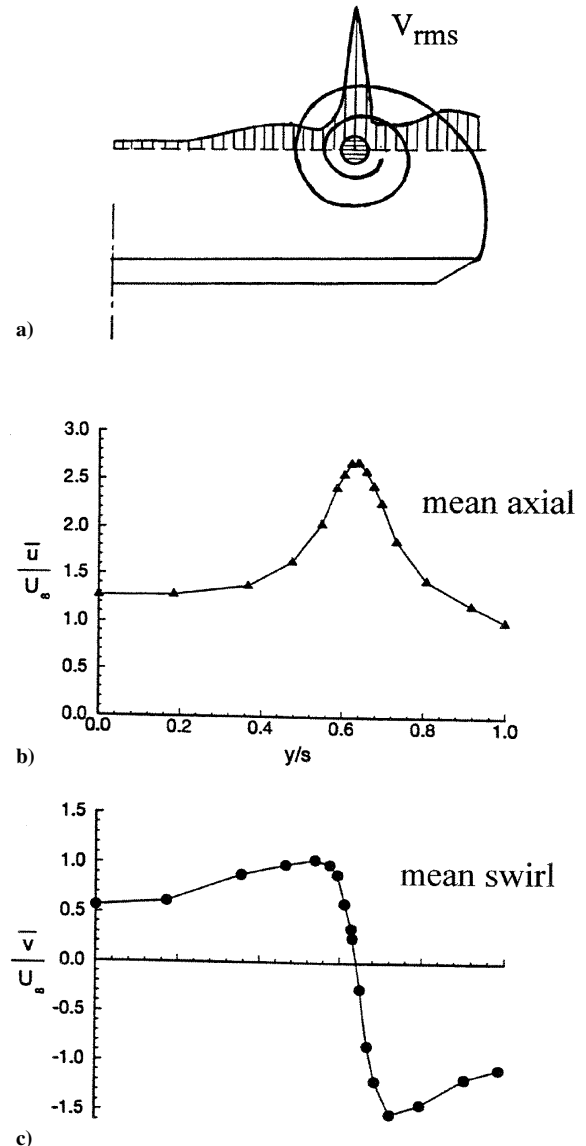


Fig. 4 Variation of a) rms swirl velocity, b) time-averaged axial and c) time-averaged swirl velocity profiles across the vortex core.

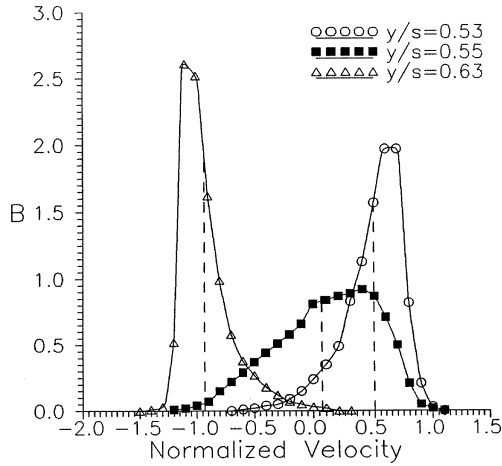


Fig. 5a Probability density functions of swirl velocity for different radial distances,  $x/c = 0.6$ ,  $\alpha = 20$  deg.

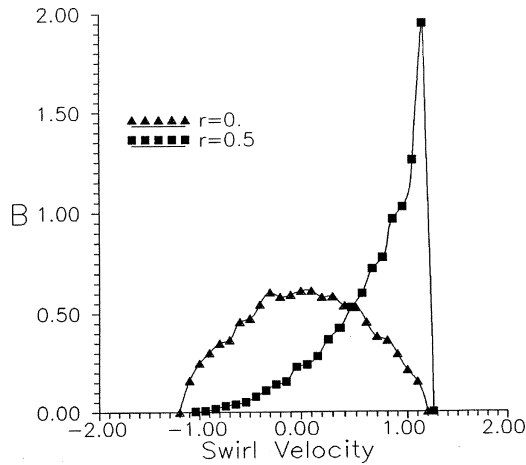


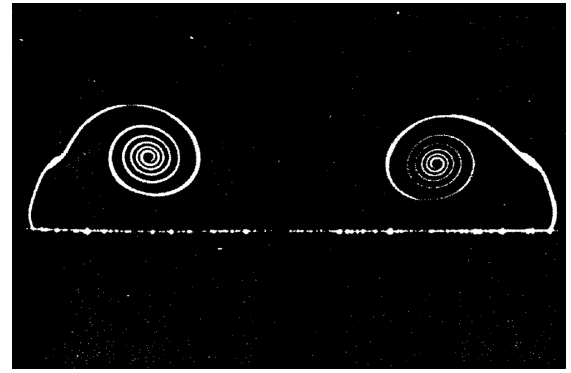
Fig. 5b Probability density functions of swirl velocity for the wandering  $Q$  vortex.<sup>21</sup>

Reynolds number is increased, the flow becomes unsteady and vortical structures appear within the separated free shear layer (Fig. 6b). The variation of the maximum value of  $v_{rms}/U_\infty$  is shown in Fig. 7 as a function of Reynolds number  $Re = U_\infty c/\nu$  (Gursul and Xie<sup>29</sup>). The two arrows in Fig. 7 indicate the corresponding Reynolds numbers of the flow visualization pictures in Fig. 6, which suggests a correlation between vortex wandering and the onset of the K-H instability. This may be explained by the Biot-Savart induction of small-scale vortices, which displace the primary vortex. Nonlinear interactions of small vortices and the primary vortex lead to random displacements of the primary vortex core. In a theoretical model of the oscillations of a side-edge vortex, Sen<sup>30</sup> showed that the trajectory of the primary vortex became chaotic due to the interaction with a small vortex.

#### IV. Vortex Breakdown

Vortex breakdown is an intriguing phenomenon that has been observed on delta wings. A flow visualization by Lambourne and Bryer,<sup>31</sup> revealing the so-called bubble-type and spiral-type vortex breakdowns is shown in Fig. 8. Subsequent research suggested that the spiral type is more common over delta wings. In fact, even the bubble type of vortex breakdown switches to the spiral type from time to time in experiments.

Different explanations of the vortex breakdown phenomenon based on hydrodynamic instability, wave propagation, and flow stagnation are summarized in several review articles.<sup>32-34</sup> It is now widely agreed that this phenomenon is a wave propagation phenomenon, and there is a strong analogy to shocks in gasdynamics.



a)



b)

Fig. 6 Flow visualization of shear layer<sup>14</sup>: a)  $Re \approx 7000$  and b)  $Re \approx 23,000$ .

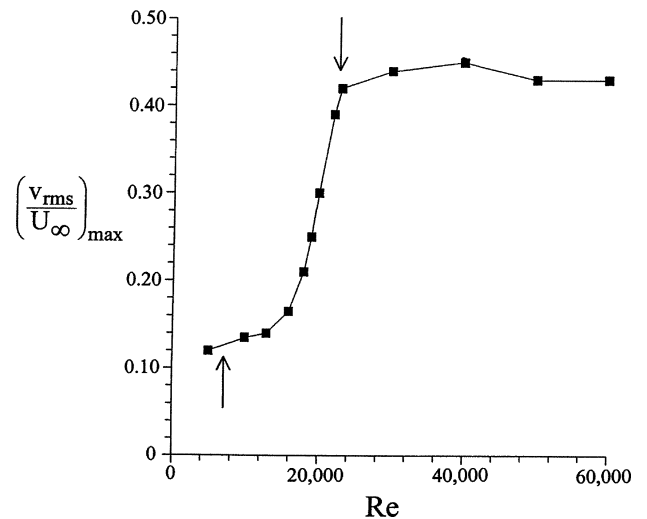


Fig. 7 Variation of maximum rms swirl velocity as a function of Reynolds number.<sup>29</sup>

Concepts of supercritical and subcritical flows based on wave propagation characteristics play an important role in the understanding of vortex breakdown.

Both experiments and theoretical explanations show that there are two important parameters affecting the occurrence and movement of vortex breakdown: swirl level and external pressure gradient outside the vortex core. An increase in either parameter promotes the earlier occurrence of breakdown. For leading-edge vortices, both parameters depend on the wing geometry, such as incidence and sweep angle.

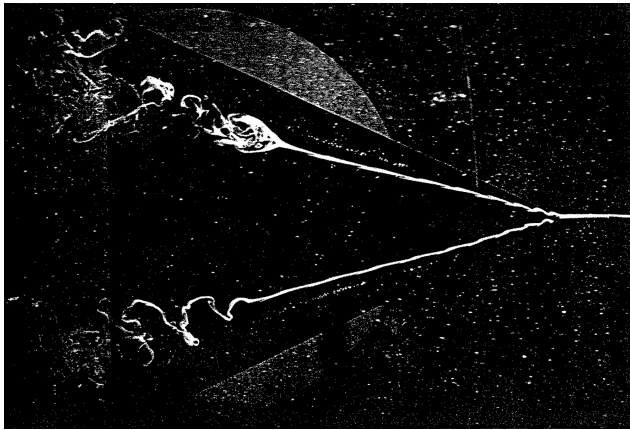


Fig. 8 Flow visualization of vortex breakdown over a delta wing.<sup>31</sup>

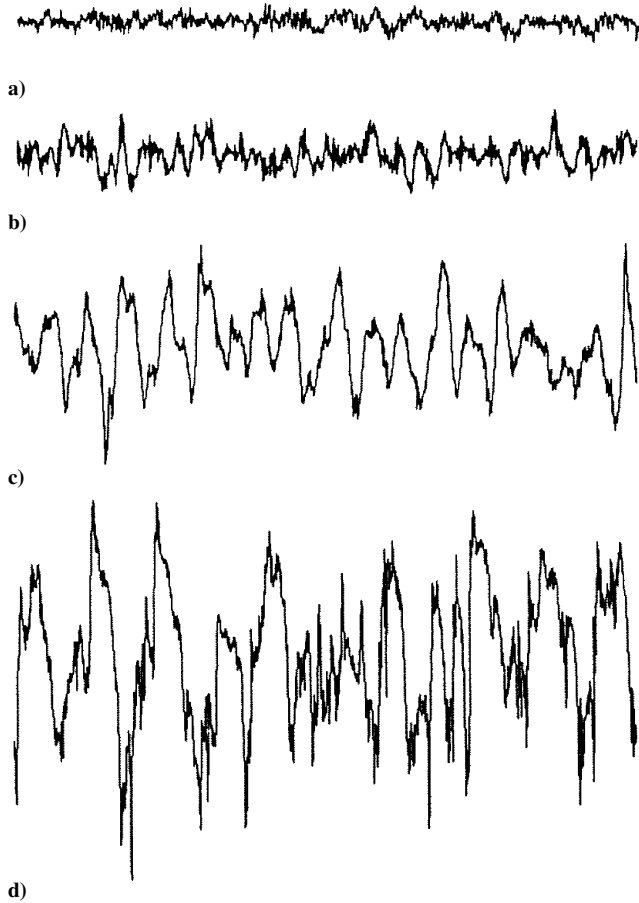


Fig. 9 Pressure fluctuations<sup>35</sup> near trailing edge ( $x/c = 0.93$ ) for angle of attack a)  $\alpha = 15$  deg, b)  $\alpha = 29$  deg, c)  $\alpha = 34$  deg, and d)  $\alpha = 39$  deg;  $y/s = 0.5$ , length of time record approximately  $10c/U_\infty$ .

#### A. Helical Mode Instability

It is well-known that the flow downstream of vortex breakdown exhibits a well-documented hydrodynamic instability. Periodic oscillations were observed in a variety of swirling flows after breakdown occurred. With regard to leading-edge vortices, examples of time histories of wing surface pressure at  $x/c = 0.93$  are shown in Fig. 9 (taken from Gursul and Yang<sup>35</sup>). For the smallest angle of attack  $\alpha = 15$  deg, the breakdown location is not over the wing. With increasing angle of attack, the amplitude of the pressure fluctuations increases as vortex breakdown moves over the wing. The quasi-periodic pressure fluctuations are clearly seen, whereas the frequency content also varies with the angle of attack.

Unsteady pressure measurements on delta wings and fins revealed downstream convection of a wave pattern associated with

vortex breakdown.<sup>36–38</sup> Experimentally observed periodic velocity/pressure oscillations<sup>36,39</sup> correspond to the most unstable normal modes of the time-averaged velocity profiles of the vortex (downstream of breakdown) based on the linearized, inviscid stability analysis. The disturbances are represented as  $\exp[i(kx + n\phi - \omega t)]$ , where  $\omega$  is the frequency,  $k$  the wave number in the axial direction, and  $n$  the wave number in the angular direction. By the use of two-point pressure measurements in the axial and spanwise directions,<sup>36</sup> it was demonstrated that these fluctuations are due to the first helical mode ( $n = 1$ ). If one considers constant phase surfaces at a certain instant, the description for a helix is obtained, that is,  $kx + \phi = \text{constant}$ , which shows that the sense of the helix is opposite to the direction of rotation in the vortex.<sup>40</sup> However, the whole structure rotates with a frequency  $\omega$  in the same direction as the vortex. Experiments also indicate that the frequency decreases in the streamwise direction, which implies that the pitch of the helix increases in the streamwise direction. This was confirmed by the instantaneous azimuthal vorticity distribution<sup>41</sup> in a plane that passes through the axis (Fig. 10). The vorticity concentrations are staggered like a Kármán vortex street and the spacing increases in the streamwise direction.

The proposed model of the helical mode instability as a helical vortex filament is consistent with observations of unsteady velocity and pressure measured by probes immersed into a breakdown wake. For example, Jaworski et al.<sup>42</sup> and Lee and Brown<sup>43</sup> showed that the rms values of velocity and pressure are at a minimum at the axis and become maximum at a certain radial distance. On the other hand, the location of the maximum rms pressure on the wing surface is directly beneath the vortex axis.<sup>36,42</sup> This can be confirmed by consideration of a simple flow model<sup>44</sup> in which an infinite helical vortex is located at a certain distance over an infinite plane.

Also, this description of the helical mode instability is consistent with well-known observations of the spiral breakdown over delta wings.<sup>31</sup> It is well-known that this is the dominant mode observed over delta wings.<sup>4,31,45</sup> It was also observed in flight<sup>46</sup> for an F-18, which indicated that it was not limited to low Reynolds numbers. Analyzing flow visualization studies in wind tunnels, Jumper et al.<sup>47</sup> indicated that even in the cases where breakdown looked like a bubble type, a spiral form was identified in instantaneous pictures with a short exposure time. This supports the view that the spiral form is a consequence of the instability of the bubble form.<sup>32</sup> For spiral breakdown, the sense of the helix is opposite to the direction of rotation in the main vortex as observed over delta wings.<sup>47</sup> However, in some vortex-tube experiments,<sup>48,49</sup> it was reported that the sense of helix is the same as the direction of swirl, whereas in some others (with tangential inlets) the opposite was found.<sup>32</sup> Leibovich<sup>34</sup> attributed this inconsistency to the differences in the way swirl is generated. However, in recent experiments in which swirl is also generated by guide vanes,<sup>50</sup> the sense of helix was found to be opposite to the direction of swirl. The sense of helix was found opposite to the direction of swirl in all swirling flows, with two exceptions,<sup>48,49</sup> and the reason behind this difference remains unknown. The opposite sense is consistent with unsteady flow in the breakdown wake. Also, Jumper et al.<sup>47</sup> pointed out in a simple model of breakdown that the sense of helix should be opposite to the direction of swirl to have axial velocities (due to the Biot–Savart induction) opposite to the freestream flow to generate stagnant flow behind the vortex breakdown.

Flow visualization at low Reynolds numbers indicates that the helical vortex filament persists for several turns before breaking up into large-scale turbulence. However, coherent pressure fluctuations were detected at downstream locations that are very far away from the breakdown location. In addition, these coherent fluctuations were detected at high Reynolds numbers<sup>51</sup> of the order of  $10^6$  as well as on full-scale aircraft.<sup>52</sup> The dominant frequencies for aircraft models also agree with the frequency of the helical mode instability.<sup>53</sup>

As discussed earlier, the pitch (wavelength) of the helical mode instability increases in the streamwise direction, whereas the frequency decreases. Measurements of pressure fluctuations at different streamwise locations on delta wings suggest that the

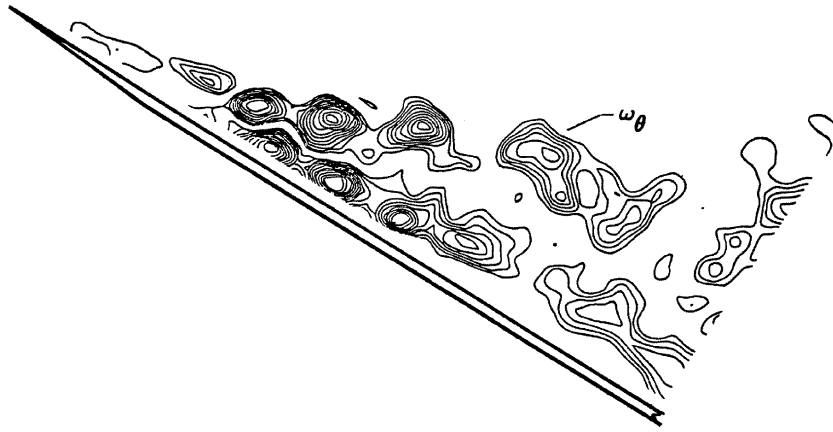


Fig. 10 Instantaneous azimuthal vorticity distribution in a plane that passes through the axis.<sup>41</sup>

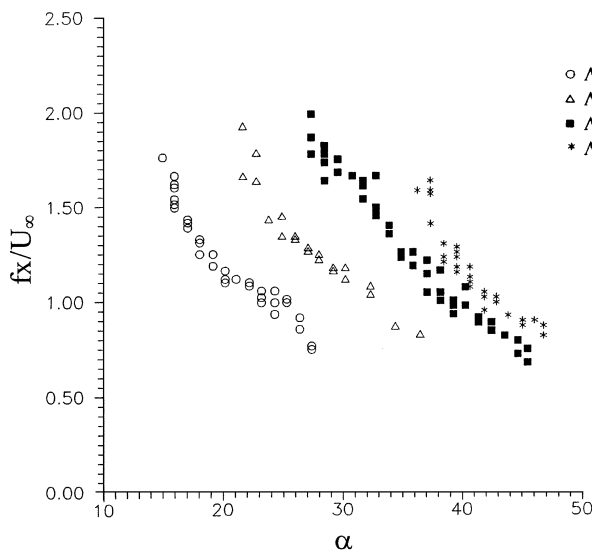


Fig. 11 Variation of dimensionless frequency as a function of angle of attack for different sweep angles.<sup>36</sup>

dimensionless frequency  $f_x/U_\infty$  is nearly constant for a given geometry.<sup>36,51</sup> Knowledge of the dominant frequency of the helical mode instability as a function of wing geometry (angle of attack and sweep angle) is important for buffeting problems, particularly fin buffeting. Figure 11 shows the dimensionless frequency  $f_x/U_\infty$  as a function of angle of attack for different sweep angles.<sup>36</sup> The range of the data is nearly the same for different wings, which suggests that a single relationship can be obtained if a unique parameter is found. Gursul<sup>36</sup> suggested that  $\Gamma/U_\infty x$ , where  $\Gamma$  is the circulation of the leading-edge vortex, is a proper dimensionless number. Figure 12 shows that all of the data collapse except for  $\Lambda = 60$  deg. Because the method used to estimate circulation is a good approximation for slender delta wings, the discrepancy for  $\Lambda = 60$  deg is believed to be due to the incapability of the method to estimate circulation. Note that the dimensionless number  $\Gamma/U_\infty x$  is related to the rate of increase of the circulation along the streamwise direction (or the rate at which vorticity is fed into the leading-edge vortex) for a conical flow. Also note that this dimensionless number provides a good correlation for the location of vortex breakdown over delta wings.<sup>54</sup>

Using the data shown in Fig. 11, Mabey<sup>55</sup> proposed a relationship in the form of  $f c \cot \Lambda \sin \alpha / U_\infty = 0.27$  for the frequency at the trailing edge, that is,  $x = c$ , which gives a useful design rule for delta wings. As summarized here, there is sufficient information on the dominant frequency of the helical mode instability, which can be used for rapid calculations. Note that these data are valid for vortex breakdown naturally occurring over slender delta wings.

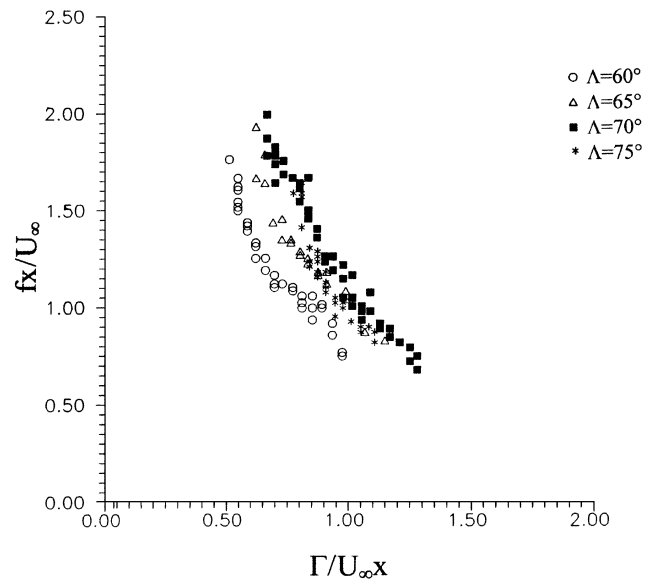


Fig. 12 Variation of dimensionless frequency as a function of dimensionless circulation.<sup>36</sup>

However, at smaller angles of attack, breakdown can be induced, for example, by the presence of a fin. There are no available data on the dominant frequencies for this type of premature vortex breakdown, which would be useful for fin-buffeting predictions.

## B. Oscillations of Breakdown Location

It was observed in several experiments that vortex breakdown location over stationary delta wings is not steady and exhibits fluctuations along the axis of the vortices.<sup>45,56</sup> Subsequently, it was discovered that these oscillations are in the form of an antisymmetric motion of breakdown locations for left and right vortices.<sup>57</sup> An example of time histories of breakdown locations for left (solid lines) and right (dashed lines) vortices are shown in Fig. 13 for  $\alpha = 37$  deg and  $\Lambda = 70$  deg (from Menke and Gursul<sup>58</sup>). The two breakdowns, which are almost mirror images, oscillate in an antisymmetric motion. The correlation coefficient in this case is  $-0.61$ . The amplitude of these fluctuations can be a significant fraction of the chord length. These oscillations may be very important for the stability and control of highly maneuverable aircraft and also have important consequences for wing and tail buffeting.

It was also reported that oscillations of breakdown locations are quasi periodic.<sup>57,59–61</sup> This coherent antisymmetric motion was clearly demonstrated by a study of the difference between the breakdown locations  $(x_{\text{left}} - x_{\text{right}})/c$  and the average breakdown location  $(x_{\text{left}} + x_{\text{right}})/2c$ . The spectra of these are shown in Fig. 14 for  $\Lambda = 75$  deg and  $\alpha = 42$  deg (from Menke and Gursul<sup>62</sup>). It is seen

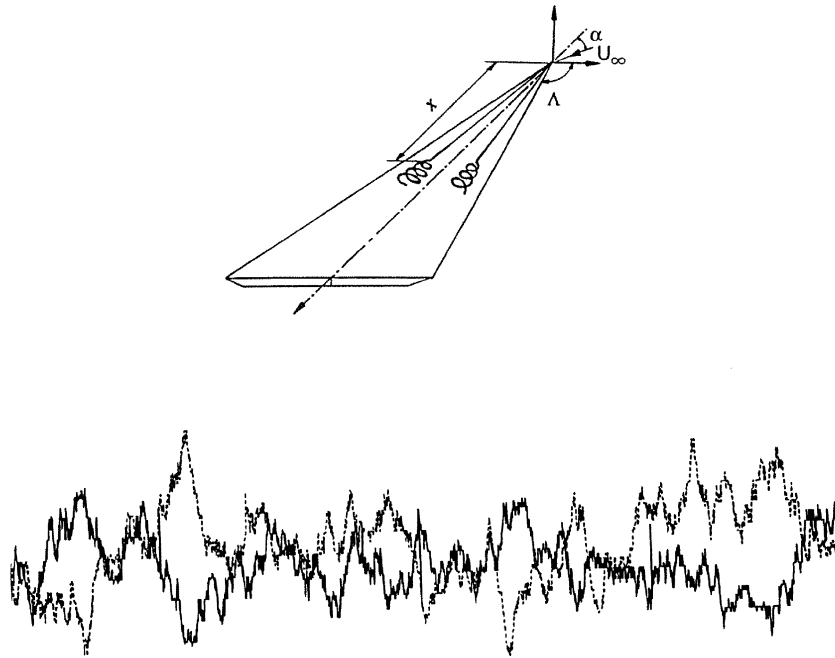


Fig. 13 Time histories<sup>58</sup> of breakdown locations for left, — and right --- vortices for  $\alpha = 37$  deg and  $\Lambda = 70$  deg.

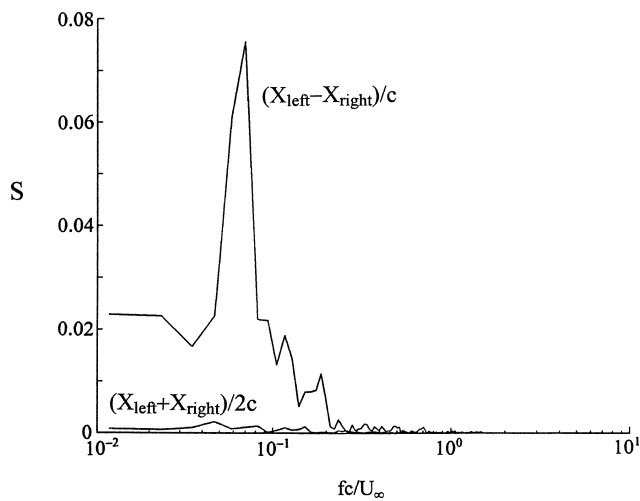


Fig. 14 Spectra<sup>62</sup> of difference and average of breakdown locations for  $\Lambda = 75$  deg and  $\alpha = 42$  deg.

that most of the energy is concentrated in the difference, and there is a dominant peak corresponding to the quasi-periodic antisymmetric oscillations.

Similar observations of quasi-periodic oscillations of breakdown location were also made by other investigators by the use of flow visualization in water tunnels. Note that these oscillations were observed at Reynolds numbers as low as  $Re = 2250$ . A similar range of dominant frequencies has been observed in all water-tunnel experiments. Recently, Mitchell et al.<sup>63</sup> carried out smoke flow visualization in a wind tunnel from  $Re = 9.75 \times 10^5$  to  $2.6 \times 10^6$ , and reported a similar range of dominant frequencies of oscillations of breakdown location. Lambert and Gursul<sup>64</sup> presented further quantitative evidence of oscillations of breakdown location at  $Re = 1.6 \times 10^6$  by using two-point surface pressure measurements. The spectra of the difference  $(p_{\text{left}} - p_{\text{right}})$  and the average  $(p_{\text{left}} + p_{\text{right}})/2$  are shown in Fig. 15. Again, it is seen that most of the energy is concentrated in the difference, which confirms that pressure fluctuations are mostly out of phase. Of course, the same conclusion could be reached by examination of the amplitude and phase of the cross spectrum of left and right pressure signals as shown by Lambert.<sup>65</sup>

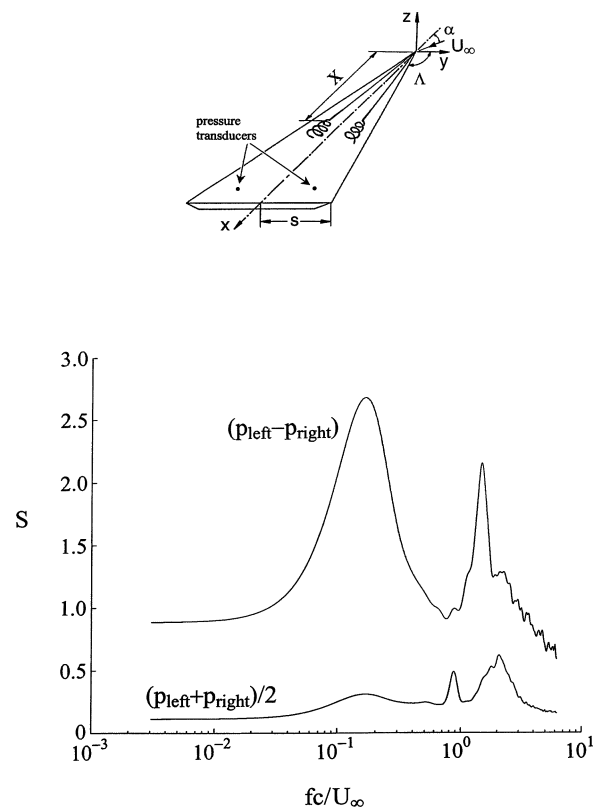


Fig. 15 Spectra<sup>64</sup> of the difference and average pressure fluctuations;  $\alpha = 50$  deg and  $\Lambda = 80$  deg.

In summary, recent investigations indicate that the phenomenon of quasi-periodic oscillations of breakdown location also exists at high Reynolds numbers.

A possible relationship between these quasi-periodic oscillations and the hydrodynamic instability of breakdown wake, that is, the helical mode instability, was investigated by Gursul and Yang.<sup>59</sup> However, it was shown that the dominant frequency of the oscillations of breakdown location occurred at much lower frequencies

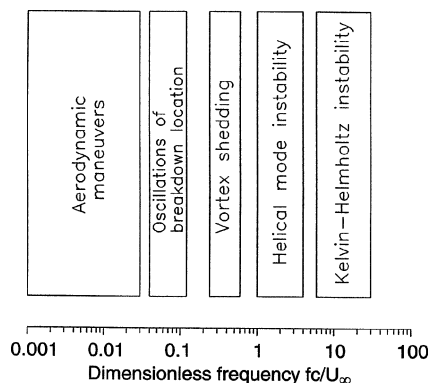
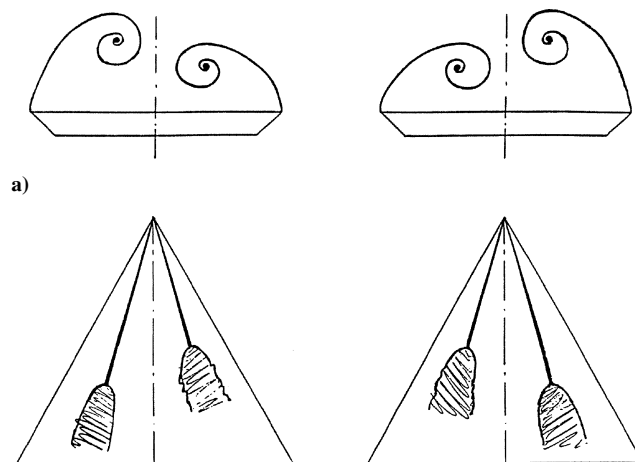


Fig. 16 Spectrum of unsteady flow phenomena over delta wings as a function of dimensionless frequency.<sup>60</sup>

than the frequency of the helical mode instability. Therefore, the helical mode instability has no effect on the oscillations of breakdown location. In fact, it was shown by Menke et al.<sup>60</sup> that the frequency of this organized motion is much smaller than the frequency of any other known instabilities. The spectrum of unsteady flow phenomena over delta wings as a function of dimensionless frequency is shown in Fig. 16. The frequency range of vortex shedding is also shown, although this phenomenon is distinctly different from other phenomena presented in Fig. 16 and is not observed until the vortex breakdown location reaches the apex. When compared with the frequency of other phenomena, the frequency range of oscillations of breakdown location for a stationary delta wing is much closer to the frequency range of typical aerodynamic maneuvers (up to  $fc/U_\infty \cong 0.03$ ). The response of breakdown location and possible coupling between the wing motion and breakdown location in this frequency range is very important.

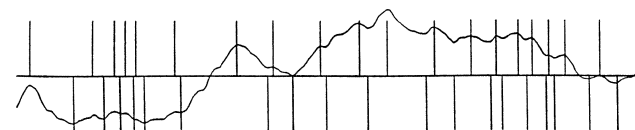
To understand the mechanism leading to oscillations of breakdown location, extensive experiments were conducted, and spectral analysis and other statistical concepts were used to quantify the unsteady behavior of vortex breakdown location obtained from flow visualization.<sup>60</sup> It was found that oscillations become larger and more coherent as time-averaged breakdown locations get closer to each other, that is, when the angle of attack or sweep angle is increased. Although these kinds of vortex interactions are more of a concern for slender wings, evidence of such interactions at a relatively low sweep angle of  $\Lambda = 60$  deg was recently reported.<sup>66</sup> Wing tip accelerations occurred in an antisymmetric structural mode for a slightly flexible delta wing when vortex breakdown occurred on the wing.

The detailed mechanism that is responsible for this interaction is not clear. One possible mechanism is that the interaction is the result of a crossflow instability (Fig. 17). This is very similar to the well-known vortex asymmetry problem in forebody flows, except that the asymmetry for delta wing vortices is time dependent, whereas the time-averaged locations of vortex cores are symmetric. The oscillations of vortex cores could be imagined to arise from a loss of stability by the symmetric mean flow. It is possible that this interaction causes the location of breakdown to vary periodically in an antisymmetric motion. Although no signs of crossflow instability were observed in flow visualization experiments, this may be due to the small amplitude of core displacements. A second possible mechanism is that the interaction is the result of a streamwise instability of breakdown regions, and vortex breakdown is a necessary part of the onset of the instability. Further studies are needed to clarify the physical mechanism of the interaction in the light of recent evidence,<sup>67</sup> which indicates that crossflow instability may exist. Also, periodic oscillations of vortex cores about mean symmetric positions have been observed in flow visualization experiments for a circular cone.<sup>68</sup> This quasi-periodic phenomenon was observed visually near the onset angle of attack of asymmetry and over a certain range of Reynolds numbers. A recent theoretical work<sup>69</sup> revealed that vortices over a thin slender delta wing are stable to all disturbances of small amplitude based on the slender-body theory for

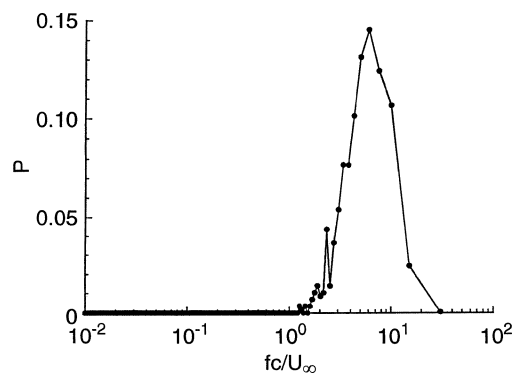


b)

Fig. 17 Possible interaction mechanisms: a) crossflow instability and b) streamwise instability.



a)



b)

Fig. 18 a) PVC algorithm and b) PVC histogram for  $\alpha = 42$  deg and  $\Lambda = 75$  deg (Ref. 60).

an inviscid incompressible flow, but that the vortices may become unstable with increasing wing thickness.

Another interesting aspect of vortex interactions was revealed by velocity measurements performed upstream of breakdown location.<sup>60</sup> The quasi-periodic velocity oscillations, which are induced by the oscillations of breakdown location through upstream influence, were observed in the primary vortex core as well as in the secondary vortex core. Hence, a secondary interaction between the primary and secondary vortices is evident. It is not known what effect this has on the unsteady aerodynamics of delta wings.

It is obvious from the time series of breakdown location that there exists much small-scale motion. The time history of breakdown location consists of low-frequency, large-amplitude fluctuations and high-frequency, low-amplitude fluctuations. As discussed earlier, the low-frequency, large-amplitude motion dominates the spectrum and is responsible for the large-scale displacements. On the other hand, the source of the small-scale motion is not clear. More information about the high-frequency fluctuations was obtained by the peak-valley-counting (PVC) technique developed for the study of the small-scale motion from velocity traces in shear layers.<sup>70,71</sup> The principle of this technique is to identify every local maximum and every local minimum due to the high-frequency fluctuations and to assign a pulse of (+1) or (-1), as shown in Fig. 18a. The interval

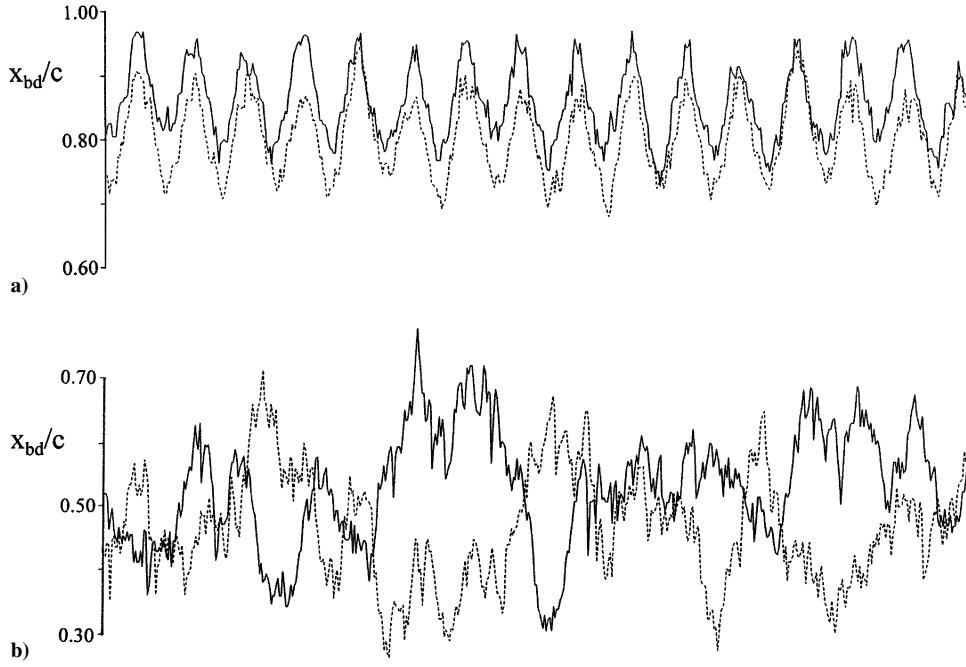


Fig. 19 Time histories<sup>62</sup> of breakdown locations for left and right breakdowns for a)  $\alpha_0 = 29$  deg and b)  $\alpha_0 = 42$  deg for  $\alpha_1 = 3$  deg,  $fc/U_\infty = 0.1$ ; length of time record is  $170c/U_\infty$ .

between successive positive (or negative) pulses represents the period of that particular event. The probability density function of the interval between successive positive pulses is shown as a histogram in Fig. 18b for  $\Lambda = 75$  deg and  $\alpha = 42$  deg. The PVC histogram shows a maximum around  $fc/U_\infty \cong 6.1$ . Similar histograms for the other test cases reveal that there exists a maximum of the probability density function for a frequency between  $fc/U_\infty = 6$  and  $fc/U_\infty = 10$ . This range of dimensionless frequency falls in the same range of frequency of the K-H instability (Fig. 16).

Vortex interactions over delta wings may become even more complex for maneuvering aircraft. Recent research has shown that the dynamic response of breakdown location to small-amplitude pitching oscillations of a delta wing may be very complex.<sup>62</sup> At moderate angles of attack, left and right breakdown locations are in phase and locked to the pitching frequency (Fig. 19). At large angles of attack, the natural antisymmetric oscillations and the symmetric oscillations caused by the pitching motion are observed simultaneously. The resulting motion is no longer periodic and a phase-averaging (ensemble-averaging) technique is not appropriate to study the variation of breakdown. There is also a remarkable antisymmetry between the left and right breakdowns. It was shown that the mode competition between the symmetric and antisymmetric modes is strongly affected by the excitation frequency. The relative amplitude of the symmetric mode increases with increasing frequency. This was demonstrated by consideration of the rms values of the difference and the average. The ratio of these rms values is defined as

$$R = \frac{(x_{\text{left}} + x_{\text{right}})_{\text{rms}}}{2(x_{\text{left}} - x_{\text{right}})_{\text{rms}}} \quad (1)$$

The variation of this ratio with reduced frequency is shown in Fig. 20. It is seen that this ratio increases with increasing reduced frequency, which confirms that the symmetric mode becomes more important with increasing reduced frequency. The effect of the reduced frequency on the mode competition can be also seen by the study of the correlation coefficient, which increases with reduced frequency. The physical mechanism of this observation remains to be explored.

The response of breakdown location to small-amplitude pitching oscillations of the delta wing showed that it has some features of a nonlinear system, such as the existence of multiple frequencies in the spectrum. Also, there are other indications that it has

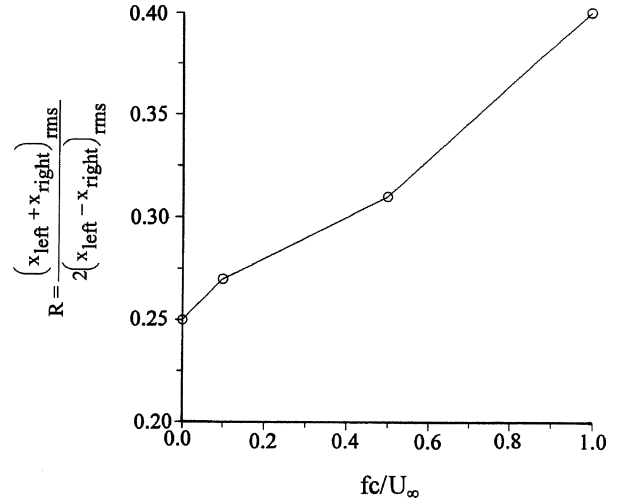


Fig. 20 Variation of the ratio  $R$  with reduced frequency,  $\alpha_0 = 42$  deg and  $\alpha_1 = 3$  deg.

some similarities to a self-excited oscillator.<sup>72</sup> For example, even very small-amplitude pitching motion is sufficient to cause a large increase in the amplitude of the antisymmetric oscillations of breakdown location compared to that of the stationary wing.

## V. Leading Edge Vortices in Unsteady Flows

Up to this point, only stationary delta wings have been considered. In this section, unsteady flow phenomena will be considered over dynamic delta wings. The earliest investigation in this direction was a flow visualization study over a delta wing oscillating in heave.<sup>73</sup> When the variation of the height of vortex core (relative to the wing surface) was studied, it was found that there existed a phase lag of the motion of the vortex core with respect to its variation in the quasi-steady case.

Lambourne et al.<sup>74</sup> investigated the transient behavior of leading-edge vortices over a plunging delta wing. This study was conducted in a water tunnel and was concerned with how vortices and separated shear layers developed after the model was driven at a constant plunge velocity, which corresponded to a sudden increase in angle

of attack from zero. An example of the variation of the height of vortex center above the wing as a function of dimensionless time is shown in Fig. 21. These results indicate that after the start of the plunge, the movement of the vortex core with respect to the wing is almost completed within one local convective time unit (the time required for the freestream to travel from the apex to the crossflow plane). Note also that the response of a vortex core is similar to that of a first-order dynamic system to a step function input. Therefore, with this model, one also expects to find a phase lag in the variation of position of vortex core, which is consistent with the findings of Maltby et al.<sup>73</sup> in oscillatory experiments.

Gad-el-Hak and Ho<sup>75</sup> conducted a flow visualization study on pitching delta wings. It was noted that leading-edge vortices executed a growth-decay cycle and that the flow patterns revealed the existence of a hysteresis loop. The variation in the height of the dyed region in the cross plane as a function of angle of attack is shown in Fig. 22. During the upstroke motion, the dyed region was thinner than that of the quasi-steady case (at a given angle of attack) due to the time delay in the development of the vortex. During the downstroke motion, the height of the dyed region again lagged behind

the quasi-steady value, which resulted in a thicker dyed region. In summary, reported phase lags and hysteresis loops indicate that a time lag in the development of the cross-sectional flow pattern exists in unsteady flows.

## VI. Vortex Breakdown in Unsteady Flows

Extensive experimental studies of the dynamic response of vortex breakdown due to wing motion have been conducted. The most important observation from these experiments was that the vortex breakdown had a time delay with respect to the quasi-steady case. Computational results<sup>76</sup> for transient vortex breakdown above a delta wing subject to a pitch-and-hold maneuver to high angle of attack showed that the onset and delay of vortex breakdown are strongly linked to the adverse pressure gradient along the vortex axis, which depends on the wing angle of attack and pitching motion.

The effect of adverse pressure gradient is also important for vortex breakdown over a stationary delta wing in unsteady freestream.<sup>77</sup> If the vortex breakdown location is away from the trailing edge and closer to the apex in steady freestream, the unsteadiness does not affect the burst position. Otherwise, the breakdown may appear suddenly at an upstream location, depending on the frequency and amplitude, as shown in Fig. 23 (Gursul and Ho<sup>78</sup>). It was suggested that the time-dependent nature of burst position is due to the relative variations in pressure gradient on the wing surface, which is expected to be more important near the trailing edge. Also note that the shape and size of vortex breakdown region/wake may be very different in unsteady flows. For example, a highly elongated shape of breakdown at  $t/T = 0$  is very unusual compared to the bubble type of breakdown at  $t/T = 0.25$ .

Abrupt transformations of the leading-edge vortex have been observed when a delta wing was pitched to high angle of attack.<sup>79</sup> These abrupt transformations are characterized by different propagation speeds of breakdown (by one order of magnitude) and different shapes of wake region. An example of contours of constant azimuthal vorticity showing abrupt transformation of structure of leading-edge vortex is given in Fig. 24. During the initial stage of pitch-up maneuver ( $t^* = 0.28$ ), the shape of the breakdown region is more like the bubble type, whereas at a later stage ( $t^* = 0.42$ ) it has a highly elongated shape. The changes in propagation speed have obvious consequences for the time lags during a maneuver. Also, changes in wake region are likely to affect the instabilities

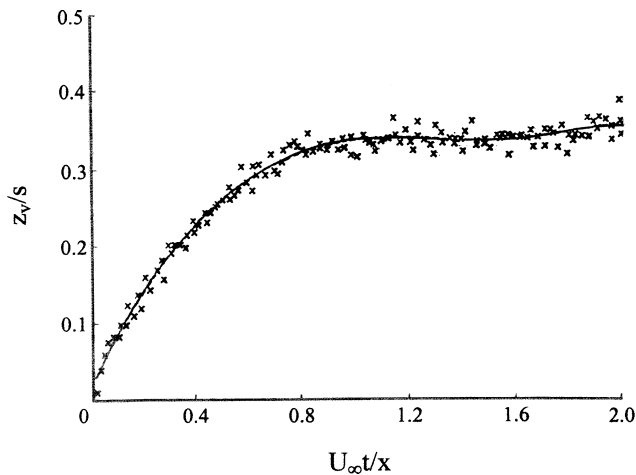


Fig. 21 Height of vortex center above the wing following a sudden change of incidence.<sup>74</sup>

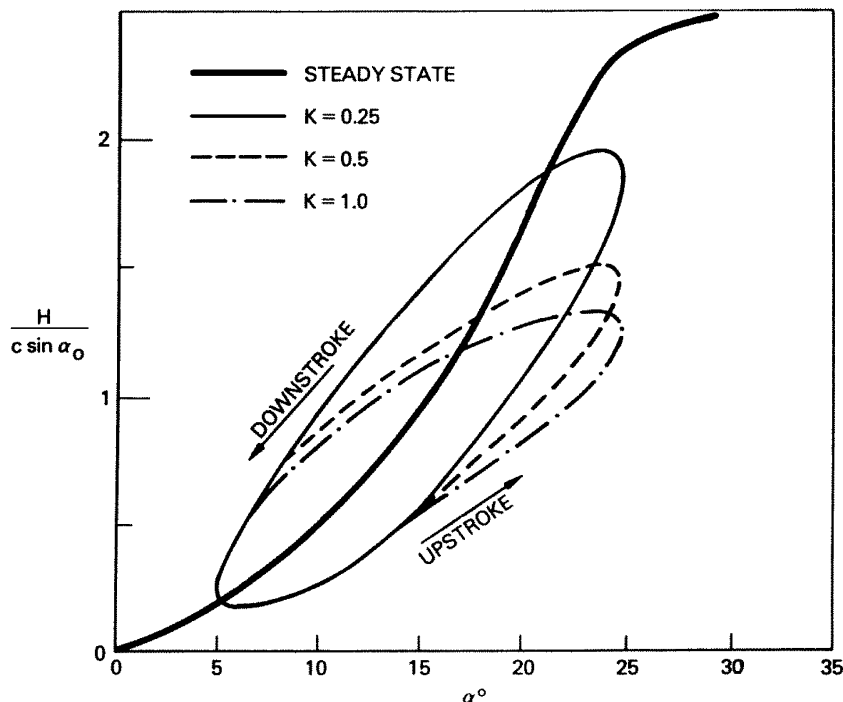


Fig. 22 Variation of height of dyed region in a crossflow plane as a function of angle attack for a periodically pitching delta wing.<sup>75</sup>

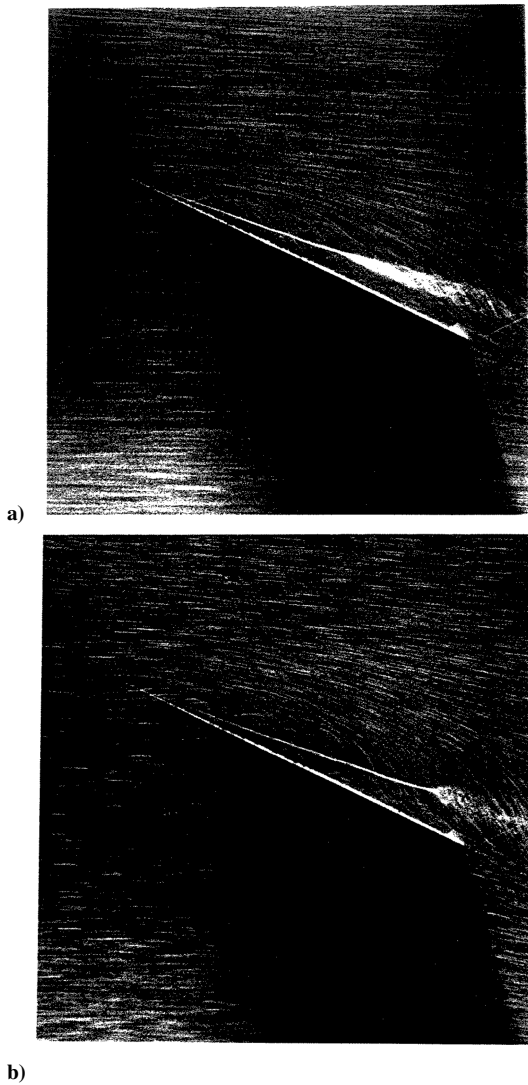


Fig. 23 Flow visualization of vortex breakdown for unsteady freestream<sup>78</sup>:  $U/U_\infty = 1 + R \cos(\omega t)$ ,  $\alpha = 25^\circ$ ,  $\omega c/2U_\infty = 1.395$ , and  $R = 0.70$ , a)  $t/T = 0$  and b)  $t/T = 0.25$ .

associated with vortex breakdown, such as the helical mode instability, as evidenced by a recent experimental investigation.<sup>80</sup>

#### A. Response of Vortex Breakdown

As discussed earlier, when a delta wing undergoes pitching motion at high angle of attack, there is a time lag of vortex breakdown location with respect to its variation in the quasi-steady case. The earliest reference to the time lag of breakdown location was made by Lowson.<sup>45</sup> Subsequently, Krishnamoorthy<sup>81</sup> and Woodgate<sup>82</sup> observed the time lag in water-tunnel and wind-tunnel experiments, respectively. Recently, more detailed observations of the phase lag were made by Wolfelt,<sup>83</sup> Atta and Rockwell,<sup>84,85</sup> and LeMay et al.<sup>86</sup> These studies revealed that, for a periodic pitching motion, vortex breakdown location forms hysteresis loops when plotted as a function of angle of attack. An example is shown in Fig. 25. The loops become wider with increasing frequency. It was also shown that the phase lag increases with increasing reduced frequency, without significant influence of the Reynolds number.

This time lag, which is important for the stability and control of aircraft, has also been observed for other types of wing motion, such as plunging and rolling. (See Greenwell and Wood<sup>87</sup> for a summary.) The response of breakdown location was also studied for transient motions such as a finite ramp pitching motion or plunging motion by Thompson et al.,<sup>88</sup> Miao et al.,<sup>89</sup> Reynolds and Abtahi,<sup>90</sup> and Magness.<sup>91</sup> Similar observations of time lag and hysteresis effects were made for pitch-up and pitch-down motions. Also, similar time

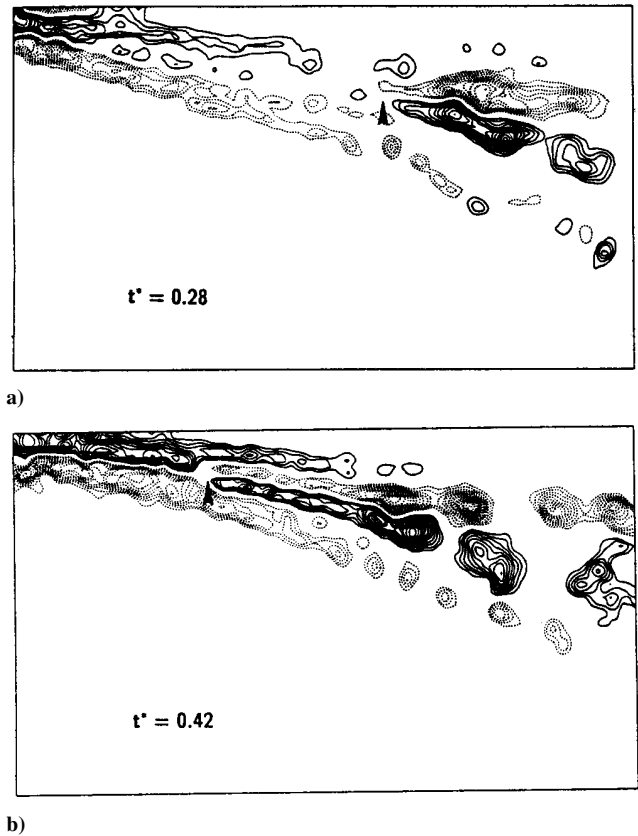


Fig. 24 Contours of constant azimuthal vorticity for transient vortex breakdown over a pitching delta wing<sup>79</sup>: a)  $t^* = 0.28$  and b)  $t^* = 0.42$ .

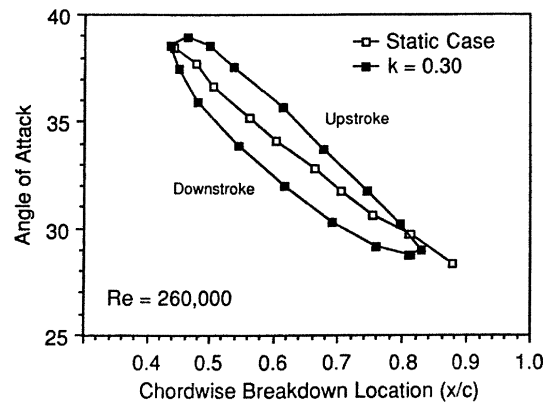


Fig. 25 Chordwise location of vortex breakdown for a pitching delta wing.<sup>86</sup>

lags were observed for a variety of wing shapes, including diamond, cropped, delta, and double delta wings.<sup>92</sup> It was commonly observed that, on completion of the wing motion, vortex breakdown required a very long time (of the order of 10 convective time units) to reach the steady-state position. Although this has been interpreted by several investigators as a separate phenomenon with a large timescale, there is no firm evidence of that, as pointed out by Greenwell and Wood.<sup>87</sup> An alternative interpretation is that the response of breakdown location is similar to that of a first-order system to a step function input, and the time constant is much smaller than 10 convective time units. For a first-order system, the output reaches the steady-state level very gradually in the form of an exponential rise, which is very similar to the response of breakdown location. With this idealization, the time constant  $\tau$  can be estimated from the time history of breakdown location in response to a given unsteady wing/surface motion. The estimated values of the time constant for different types of motion are given by Srinivas et al.<sup>40</sup> The normalized time constant  $\tau U_\infty/c$ ,

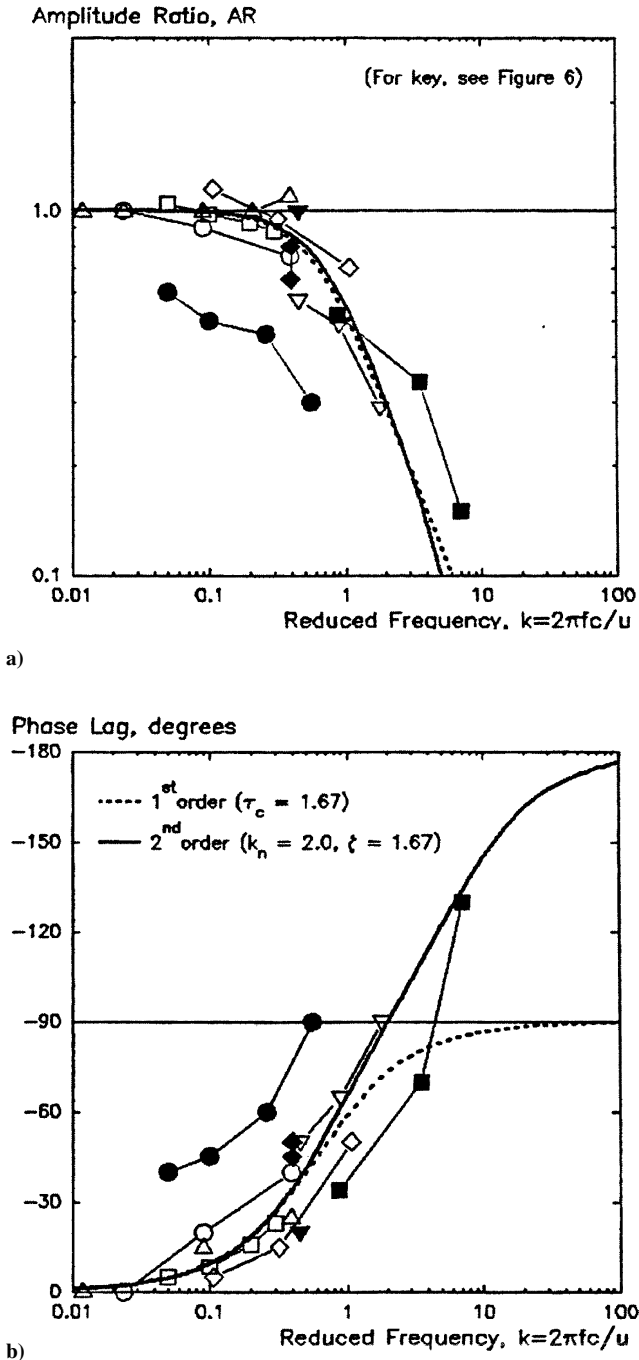


Fig. 26 Vortex breakdown location as a function of reduced frequency<sup>87</sup> for a) amplitude ratio and b) phase lag.

which is of the order of unity, depends on the type and amplitude of the motion, the breakdown location in the static case, and the sweep angle of the wing. For  $\Lambda \geq 70$  deg, the normalized time constant is around  $\tau U_\infty/c = 1-2$ . For lower sweep angles, the time constant is larger. Greenwell and Wood<sup>87</sup> modeled the dynamic response of the breakdown location by a first-order, as well as a second-order system. By curve fitting to the experimental values of the phase lag for pitching wings, they obtained  $\tau U_\infty/c = 1.67$  for a first-order system (Fig. 26). Reisenthel et al.<sup>93</sup> studied the dynamic response of breakdown location to an oscillating fin over a delta wing. They indicated that the time constant associated with the upstream motion of the vortex breakdown location is larger than that associated with its downstream motion. It was suggested that dynamic hysteresis of vortex breakdown location can be explained by the discrepancy in the time constants.

Recent investigations of vortex breakdown control techniques revealed similar time lags. The measured phase lag of breakdown

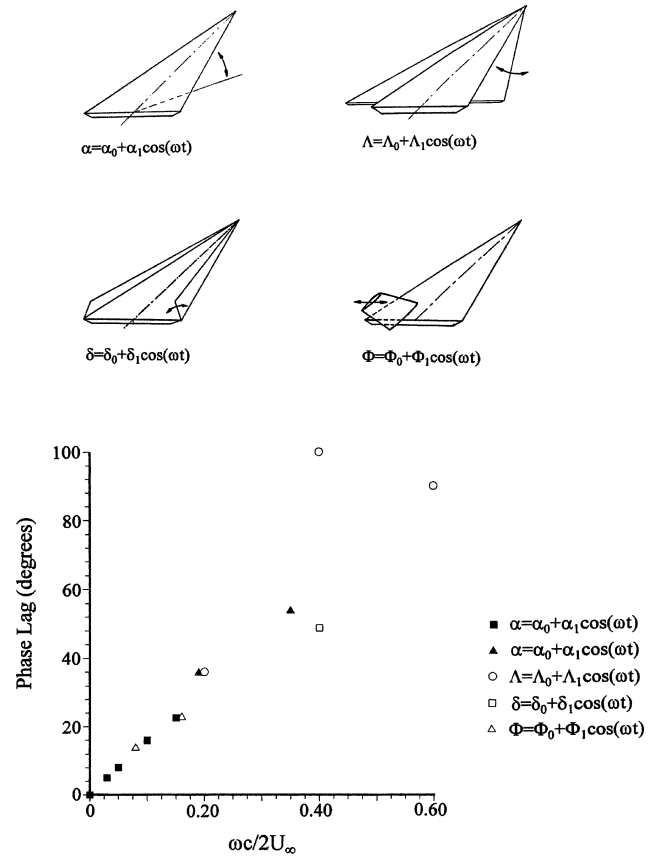


Fig. 27 Phase lag of vortex breakdown location for different types of unsteady motion.<sup>96</sup>

location with respect to the quasi-steady case is shown in Fig. 27 as a function of the reduced frequency  $K = \omega c/2U_\infty$  for oscillating leading-edge flaps<sup>94</sup> and leading-edge extensions<sup>95</sup> together with pitching wings.<sup>35,86</sup> Although there is a larger scatter of data at high frequencies, there is a consistent trend of increasing phase lag with increasing reduced frequency. Several factors are important for phase lag and contribute to the data scatter: the breakdown location in the static case, amplitude of motion, fluctuations of breakdown location that are also observed for stationary wings, the number of cycles used for phase averaging, and the method used to calculate phase lag. In particular, at large frequencies, the amplitude of the phase-averaged variations of breakdown location becomes smaller. The calculated phase lag is sensitive to the number of cycles used for phase averaging. Keeping these factors in mind, collapse of data is not expected for different motions. The purpose of Fig. 27 is simply to show that the phase lags are similar for  $\alpha$ ,  $\Lambda$ , and  $\delta$  variations.

Also shown in Fig. 27 is the phase lag of breakdown location for an oscillating fin placed near the trailing edge of a delta wing.<sup>93,96</sup> Note that the phase lag agrees very well with that of pitching wings. The effect of fin oscillations  $\Phi(t)$  is expected to be different than that of the other types of motion (shown in Fig. 27) for which the development of the leading-edge vortex is time dependent. The flow upstream is steady (in the sense of being free of periodic perturbations due to the oscillating fin) in this case. Yet, the measured phase lag due to the oscillating fin is very similar. This is not the only example where the wing is stationary, and yet a time lag of breakdown is observed. Parmenter and Rockwell<sup>97</sup> investigated the transient response of breakdown location to suction applied through a probe located well downstream of the breakdown location. Time delays and hysteresis of breakdown location were found. Another example is a stationary delta wing placed in an unsteady freestream.<sup>77</sup> Large time-dependent variations in breakdown location were observed, depending on the angle of attack. In summary, whether the unsteadiness is due to pitching, plunging, rolling, oscillating flaps/surfaces/fins, or time-dependent suction, similar phase lags have been found as a

function of frequency. This suggests that the mechanism of time lag with respect to the quasi-steady case is universal, regardless of the type of unsteady motion.

### B. Mechanism of Time Lag

There have been different suggestions as to the origin of the time lag of breakdown location in unsteady flows. In this section, these different explanations will be critically reviewed. The first possibility was that the observed phase lag of breakdown location was due to a time lag in the development of vortex flow. Indeed, the investigations by Maltby et al.,<sup>73</sup> Lambourne et al.,<sup>74</sup> and Gad-el-Hak and Ho<sup>75</sup> established that there exists a phase shift in the development of the cross-sectional flow pattern in the absence of vortex breakdown. However, this time lag of vortex development is very small compared to the large time lag of breakdown location, as indicated by Rockwell<sup>5</sup> and Ericsson.<sup>98</sup> For example, the data shown in Fig. 21 suggest that the normalized time constant associated with the movement of vortex core is around  $\tau U_\infty/c \cong 0.35$ . On the other hand, Greenwell and Wood<sup>87</sup> obtained  $\tau U_\infty/c = 1.67$  associated with the movement of vortex breakdown location over pitching wings. Also, Greenwell and Wood<sup>87</sup> presented a comparison of the time constants (estimated from the surface pressure changes in response to blowing) in the absence and presence of vortex breakdown, which are different by one order of magnitude, as shown in Fig. 28. Also, if the case of the oscillating fin (shown in Fig. 27) is considered, the upstream flow is free of unsteady effects, and there is no time lag in the development of vortical flow, yet, there is a considerable time lag of the vortex breakdown location.

For a pitching delta wing, the observed time lag of breakdown location was explained by the variations of the effective angle of attack and motion-induced longitudinal camber.<sup>98,99</sup> Both effects are due to wing motion and fail to explain the phenomenon of the time lag of breakdown location when the wing is stationary. Explanations of the time lag of breakdown location with concepts that are based on wing motion (such as effective angle of attack and longitudinal camber) are not possible for many other cases where the wing is stationary.

Vortex breakdown location is determined by two important parameters: swirl angle (or vortex strength) and external pressure gradient outside the vortex core. Hence, the variation of these parameters upstream of breakdown<sup>35,95</sup> was considered in an attempt to explain the time lag of breakdown location for a pitching delta wing. These investigations indicate that the variation of swirl angle has a very small phase lag, whereas the variation of the external pressure gradient has a comparable phase lag to that of breakdown location. Hall<sup>33</sup> showed that small external pressure gradients can be amplified along the core of the vortices, leading to a stagnation point. Thus, the large sensitivity of the breakdown location to a streamwise pressure gradient along the exterior of the vortex is very much expected. For other types of motion (other than pitching), the variation of external pressure gradient has not been investigated in relation to vortex breakdown. However, velocity measurements<sup>77,95</sup> show that

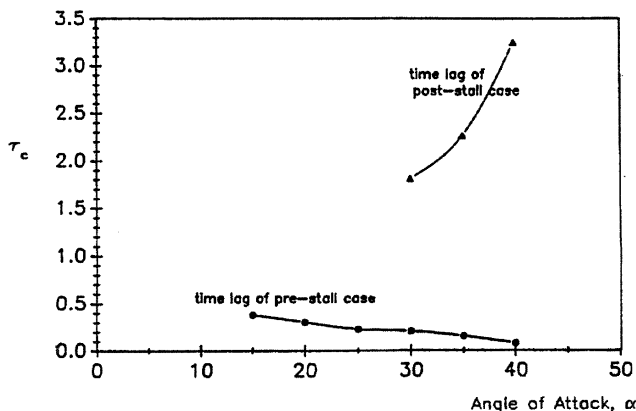


Fig. 28 Time constants estimated from surface pressure changes in response to blowing.<sup>87</sup>

the variation of swirl angle has a small phase lag, which suggests that the external pressure gradient plays a major role in the dynamic response of breakdown location.

Unlike the other cases shown in Fig. 27, in which both the external pressure gradient and the strength of the vortex are simultaneously changed as a result of moving surfaces/geometry, the case of the oscillating fin is fundamentally different. The swirl angle upstream of breakdown is constant, and, therefore, is not a variable. Yet, the vortex breakdown location is time dependent because of the time-dependent external pressure gradient set by the oscillating fin. The observed time lags and hysteresis of the vortex breakdown location are presumably due to the unsteady behavior of the external pressure gradient because it is the only dynamic parameter. Although this and other examples point toward a suspected relationship between the external pressure gradient and breakdown location, it is not clear whether this is a universal explanation of phase lag of breakdown location. Also, it is not clear how different types of motion or unsteadiness can generate similar phase lags of external pressure gradient.

Gursul<sup>96</sup> has proposed an explanation of the time lag, which is based on the theory of vortex breakdown as a wave propagation phenomenon and is universally applicable to slender vortex flows. A stationary breakdown can be considered as the superposition of an upstream moving wave and a uniform freestream velocity (in the downstream direction) that makes the wave stationary. In the dynamic case, wave speed depends on the axial wave number. For example, for a cylindrical vortex with Rankine velocity distribution and no axial velocity, the exact dispersion relation is given by Kelvin, and the speed of the waves traveling upstream can be found numerically (see Ref. 100). The speed of the waves traveling upstream decreases with increasing wave number (or frequency). As a result, the equilibrium location (vortex breakdown location) is different in the dynamic case compared to the quasi-steady case. This model predicts phase lags that increase with increasing frequency, which is well known from experimental observations for a variety of unsteady flows.

## VII. Vortex Shedding

Based on two-point velocity measurements in the wake of a delta wing, Rediniotis et al.<sup>101,102</sup> showed that vortex shedding occurs at large angles of attack. It was suggested that both symmetric and antisymmetric modes of shedding existed (Fig. 29). They conducted

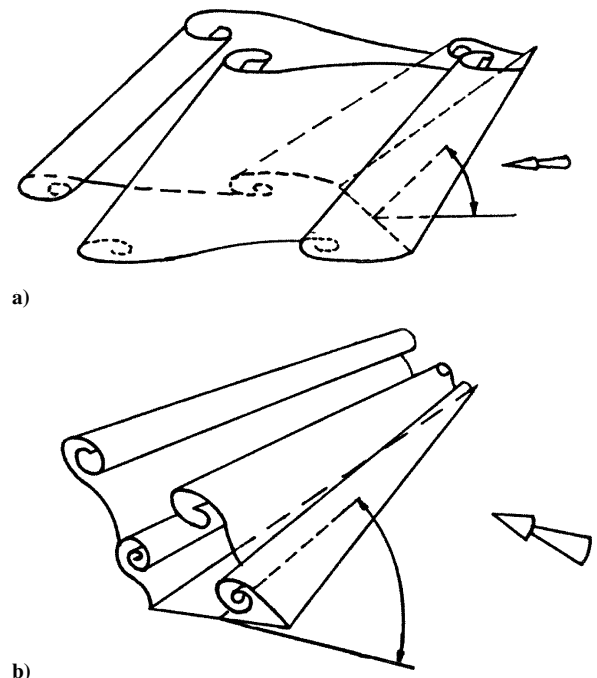


Fig. 29 Vortex shedding from delta wings<sup>101</sup>: a) symmetric and b) antisymmetric modes.

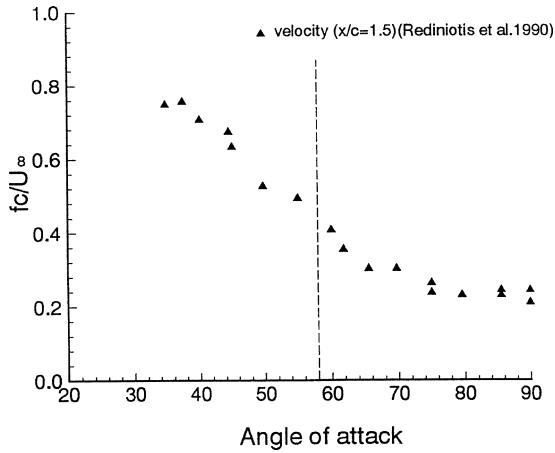


Fig. 30a Dominant frequencies in the wake for  $\Delta = 76$  deg.

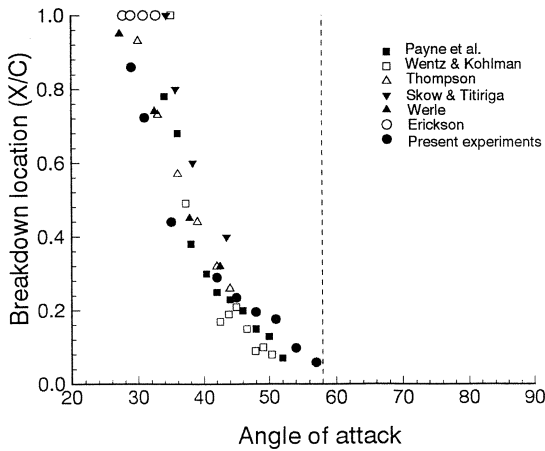


Fig. 30b Breakdown location over delta wings for  $\Delta = 75$  deg (from Ref. 103).

experiments on several delta wings with a sweep angle of  $\Delta = 76$  deg. It was shown that this quasi-periodic phenomenon occurs over a wide range of angles of attack, without a significant influence of Reynolds number. In the range of  $Re = 3.9 \times 10^4$  to  $9.02 \times 10^5$ , there was no noticeable effect on the dominant frequency of the vortex shedding. They suggest that up to  $\alpha = 70$  deg, only the symmetric mode of vortex shedding occurs. At angles of attack larger than 70 deg, both shedding modes exist simultaneously, although the symmetric mode is more dominant. Because the vortex shedding involves an interaction between the shear layers, wingspan was used as a characteristic length in the definition of the dimensionless frequency.

The dominant frequencies of velocity fluctuations measured in the wake (at  $x/c = 1.5$ ) by Rediniotis et al.<sup>101</sup> were used to calculate the conventional definition of dimensionless frequency ( $fc/U_\infty$ ) to compare with frequencies of other unsteady phenomena (Fig. 30a). Also, shown in Fig. 30b is the variation of breakdown location over delta wings with  $\Delta = 75$  deg, for which reported values from the literature are used.<sup>54</sup> The angle of attack at which breakdown location reaches the apex was estimated to be between  $\alpha = 58$  deg and 60 deg by extrapolation of data. It is clear that the data shown in Fig. 30a include vortex breakdown over the wing (at angles of attack up to 60 deg), as well as vortex shedding (at larger angles of attack). It was discussed in Sec. IV. A that vortex breakdown over the wing corresponds with the helical mode instability. Figure 30a also suggests that the dominant frequencies of the helical mode instability and those of vortex shedding form a continuous curve. This rather interesting result, which can be interpreted as a smooth transition from one phenomenon to the other, is misleading because the helical mode instability has locally varying properties.

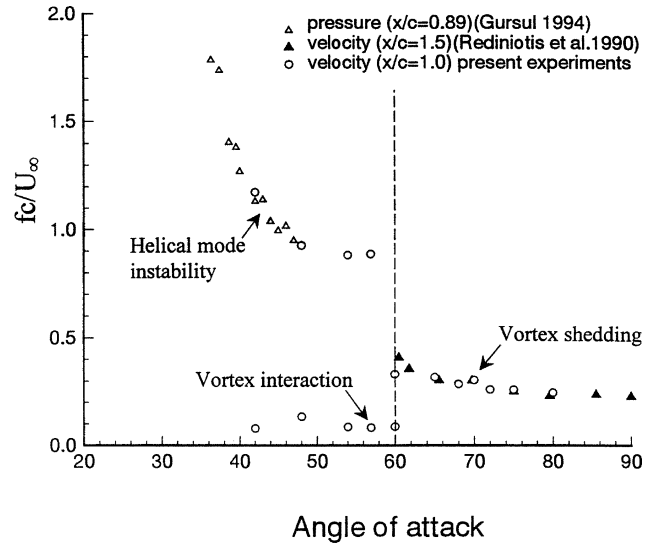


Fig. 31 Variation of dimensionless frequency for unsteady phenomena as a function of angle of attack.<sup>103</sup>

Detailed investigation of this transition was carried out by Gursul and Xie.<sup>103</sup> Measurements of the streamwise velocity at the trailing edge ( $x/c = 1.0$ ) and pressure on the wing surface ( $x/c = 0.89$ ) indicate that the helical mode instability of swirling flow disappears after vortex breakdown reaches the apex and that the dominant frequency of the vortex shedding appears in the spectra. The transition from the helical mode instability to the vortex shedding was found to be abrupt, as indicated by a jump in the frequency parameter, and occurred at the angle of attack at which breakdown reached the apex (Fig. 31). When the source of the quasi-periodic oscillations is the helical mode instability (for  $\alpha < 60$  deg), the measured frequencies at  $x/c = 1.0$  are much larger than those in the wake at  $x/c = 1.5$  (cf. Fig. 30a). This was explained by the significant changes in the wavelength of the helical mode instability in the streamwise direction in the near wake. On the other hand, when the source of the quasi-periodic oscillations is the vortex shedding (for  $\alpha > 60$  deg), the measured frequencies at  $x/c = 1.0$  and in the wake  $x/c = 1.5$  agree very well. This implies that the frequency of the vortex shedding is nearly constant in the near wake.

## VIII. Wing and Fin Buffeting

Buffeting is defined as the structural response of aircraft structures (such as wing, fin, tail, flap, and rudder) due to unsteady flow.<sup>104</sup> Several unsteady flow phenomena may excite different structural modes depending on angle of attack and freestream velocity and cause severe structural fatigue damage.

The measurements of buffeting on a slender delta wing model were reported by Mabey<sup>38</sup> (Fig. 32a). When the vortex breakdown location moved across the trailing edge (for  $\alpha \approx 20$  deg), buffeting increased very rapidly, as seen in Fig. 32a. Also, sharply increased fluctuations in the normal force coefficient for delta wings were observed when vortex breakdown moved over the wing.<sup>105</sup> Hence, the most important source of wing buffeting is the vortex breakdown phenomenon. However, light buffeting is observed in the absence of breakdown and has been attributed to vortex wandering.<sup>44</sup> Figure 32b shows the variation of wing-tip acceleration for a slightly flexible delta wing (with a sweep angle of  $\Delta = 60$  deg) with angle of attack.<sup>66</sup> It was found that maximum rms buffeting occurs when vortex breakdown is close to the apex of the wing. (Breakdown reaches the apex at around  $\alpha \approx 32$  deg.) The rms acceleration drops very rapidly in the vortex shedding regime at higher angles of attack.

Vortex breakdown is also the major source of fin buffeting in many cases. Distortion of incident vortex and interaction of vortex breakdown with fins have been investigated in detail.<sup>106,107</sup> A review of vortex/surface interactions is given by Rockwell.<sup>108</sup> Measurements of unsteady surface pressure, surface acceleration, and strain on fins

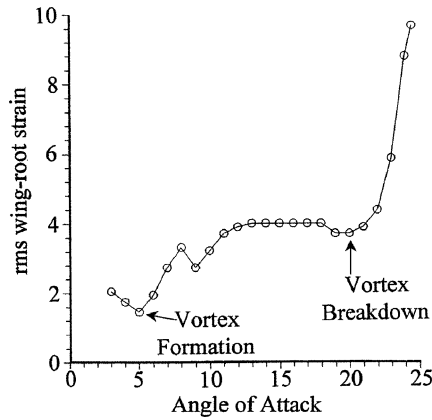


Fig. 32a Buffeting on a slender delta wing model.<sup>38</sup>

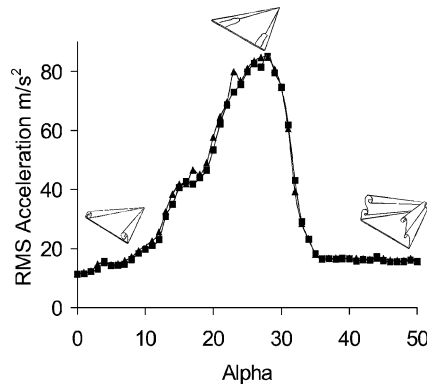


Fig. 32b Wing-tip acceleration on a slightly flexible delta wing,<sup>66</sup>  $\Lambda = 60$  deg.

generally showed that spectra had a dominant peak at high angle of attack due to the helical mode instability. Wolfe et al.<sup>109</sup> reviewed a wide variety of investigations of both simplified fin-delta wing configurations and fins on actual model aircraft. They demonstrated that the shapes of the surface pressure spectra are similar and that the dominant frequencies obey a simple scaling law over a wide range of Reynolds numbers ( $10^4$  to  $10^6$ ). It was shown that the dimensionless frequency  $f x_{bd}/U_\infty$ , where  $x_{bd}$  is the distance of breakdown location from the origin of the vortex, is roughly constant (with a value around unity). Wolfe et al.<sup>109</sup> also show that dominant frequencies of surface pressure fluctuations on fins agree very well with those on the surface of delta wings<sup>36</sup> without a fin. The dominant frequencies for aircraft models also agree with the frequency of the helical mode instability.<sup>53</sup>

An important feature of the spectra of buffeting is the existence of a low-frequency peak. For several vortex–fin interactions, a low-frequency peak was observed in the spectra of fin surface pressure, tip acceleration, strain, and velocity fluctuations around the fin.<sup>43,110,111</sup> Also, a low-frequency component in the pressure/velocity spectra was observed in the wake of breakdown in several investigations<sup>42,51,53,59</sup> in the absence of a fin. It was suggested by Wolfe et al.<sup>111</sup> that this low-frequency peak is due to the fluctuations of breakdown location in the streamwise direction. Recent investigations<sup>60</sup> show that vortex breakdown location may exhibit large fluctuations along the axis of vortices. Moreover, the quasi-periodic oscillations due to vortex interactions may be an important source of buffeting at high angles of attack.<sup>44</sup> Another possible source of buffeting is the coupling between the unsteady flow separation over the fin surface and vortex breakdown,<sup>103</sup> which was originally suggested by Patel and Hancock.<sup>112</sup> and Gordnier and Visbal.<sup>113</sup> Studies of pressure fluctuations on the fin surface<sup>111,113</sup> revealed low-frequency components that agreed well with the dominant frequencies in the spectra of the fluctuations of breakdown location. These quasi-periodic oscillations of breakdown location, which are not observed for nonimpinging flows at low angle of

attack, are an indication of a feedback effect on vortex breakdown and may play an important role in the buffeting of fins. Note that the disturbances generated at the leading edge of the fin may propagate upstream in the subcritical flow downstream of vortex breakdown.<sup>34</sup>

Also, disturbances due to aeroelastic effects (surface deflections) may propagate upstream, resulting in large oscillations of the breakdown location. This aspect was investigated in detail by Gursul and Xie.<sup>114</sup> For excitation frequencies lower than a cutoff frequency (around  $f_c/U_\infty \approx 0.40$ ), the response of vortex breakdown location is quasi periodic, and the amplitude of the variations of breakdown location decreases with increasing frequency (Fig. 33). For excitation frequencies higher than the cutoff frequency, vortex breakdown does not respond to fin oscillations. The amplitude ratio, which was defined as the ratio of the rms value of the fluctuations of breakdown location to its quasi-steady counterpart, is shown in Fig. 34 as a function of dimensionless frequency for different fin locations on the wing. It is seen that the amplitude ratio decreases with increasing frequency and that the amplitude attenuation is similar to that of a lowpass filter. It is also seen that the amplitude response is very similar for pitching delta wings and oscillating fins. In a recent investigation,<sup>115</sup> it was found that vortex breakdown had a similar response to an oscillating body in the wake. A proposed mechanism based on the wave propagation characteristics of vortex flows was presented by Gursul and Xie.<sup>114</sup> It is based on the concept of subcritical flow that exists downstream of breakdown location. Disturbances due to fin deflections may propagate upstream, and the group velocity of the disturbances depends on the axial wave number. The ability of disturbances to propagate upstream

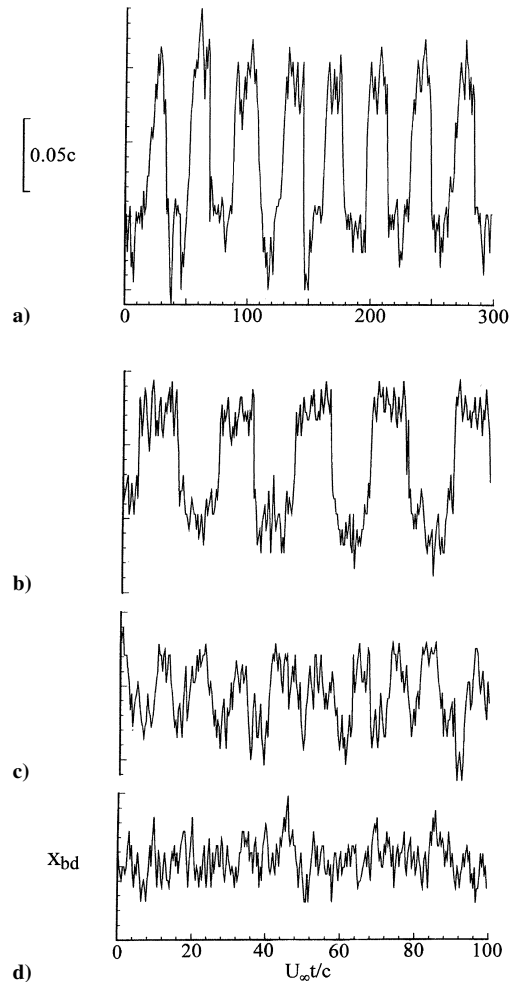


Fig. 33 Time histories of breakdown location for different fin oscillation frequencies<sup>114</sup> for a fin location of  $y_f/s = 0.8$ : a)  $f_c/U_\infty = 0.025$ , b)  $f_c/U_\infty = 0.05$ , c)  $f_c/U_\infty = 0.1$ , and d)  $f_c/U_\infty = 0.4$ .

decreases with increasing wave number (or frequency). A simple model predicts that disturbances with frequencies higher than a cut-off frequency will not propagate upstream, which agrees very well with the experimental observations.

Bean et al.<sup>116</sup> demonstrated that buffeting of centerline fins is possible at very large angles of attack where leading-edge vortex flow no longer exists. Figure 35a shows the rms pressure fluctuations on the fin as well as the buffeting response as a function of angle

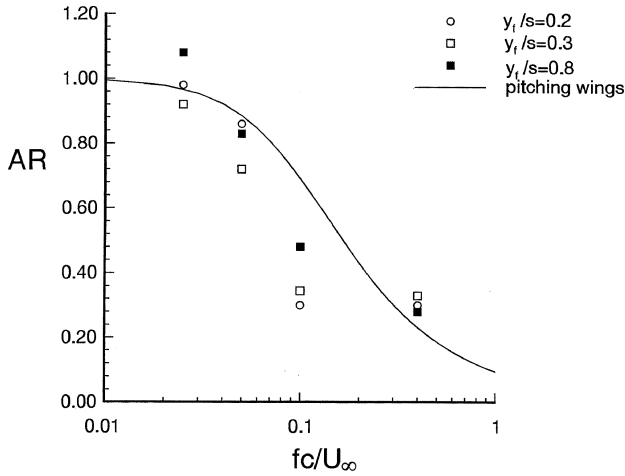
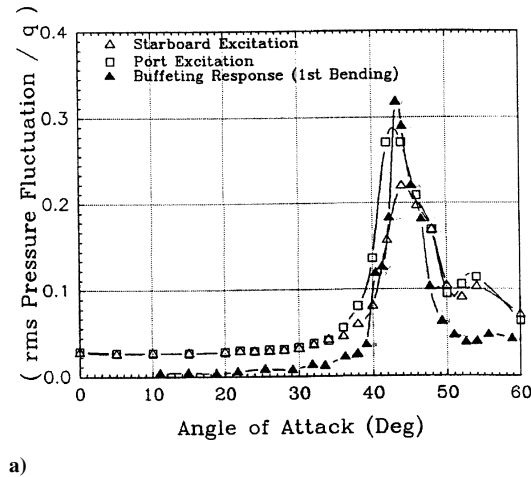
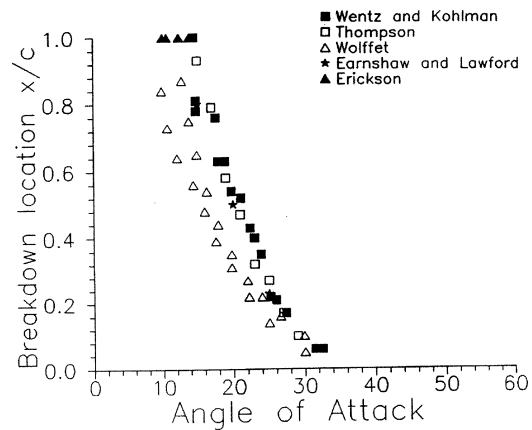


Fig. 34 Variation of the amplitude ratio as a function of forcing frequency.<sup>114</sup>



a)



b)

Fig. 35 For  $\Lambda = 60$  deg: a) rms pressure on a fin<sup>116</sup> and b) breakdown location.<sup>117</sup>

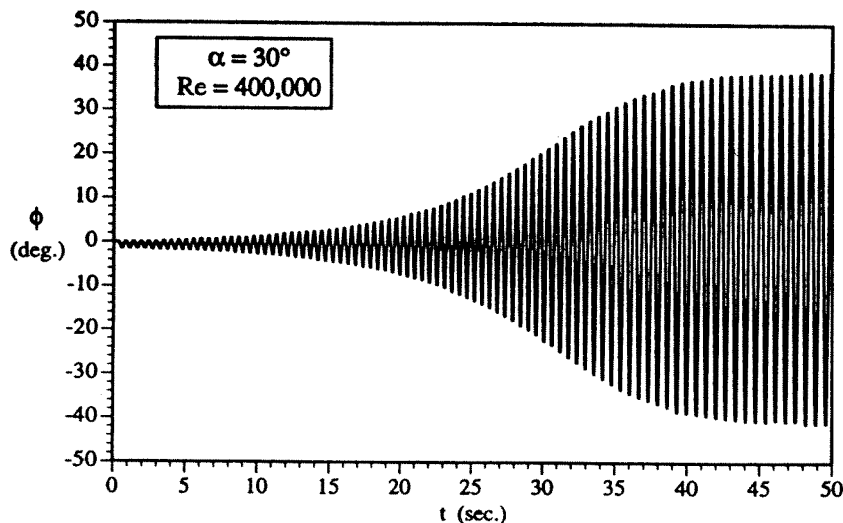


Fig. 36 Wing-rock time history for  $\alpha = 30$  deg showing buildup.<sup>119</sup>

of attack. Their delta wing had a sweep angle of  $\Lambda = 60$  deg. By using reported breakdown locations from the literature<sup>117</sup> as shown in Fig. 35b, one expects the breakdown location to reach the apex of the wing around  $\alpha \approx 32$  deg. Bean et al. showed that, for both rigid and flexible fins, the unsteady loading starts to increase at around  $\alpha = 34$  deg and reaches a sharp peak at  $\alpha = 43$  deg. It was shown that unsteady pressure on the fin surface exhibited quasi-periodic behavior. It is interesting that the buffeting was small for  $\alpha < 34$  deg, that is, vortex breakdown did not excite the fin because the fin was located at the wing centerline near the trailing edge. Therefore, the fin was away from the vortex axis. Instead, buffeting occurred in the poststall region where the vortex shedding from the wing takes place.

## IX. Wing Rock Phenomenon

Wing rock<sup>118</sup> is a self-induced limit-cycle roll oscillation that has been observed for slender delta wings as well as aircraft configurations. In particular, it has been observed in the subsonic, high angle-of-attack regime, where the leading-edge vortices are one of the most important features of the flow. An example of the time history of the roll angle is shown for  $\alpha = 30$  deg for a  $\Lambda = 80$  deg delta wing<sup>119</sup> in Fig. 36. As the angle of attack is increased, the wing rock motion first appears at a specific incidence, known as the onset angle of attack. The amplitude of the roll oscillations strongly depends on the angle of attack. An example of the measured wing-rock amplitude<sup>120</sup> is shown in Fig. 37 as a function of angle of attack. A particularly important observation is that the onset angle of attack, at which wing rock starts, decreases as the wing platform becomes more slender. This indicates a possible relation between the onset of

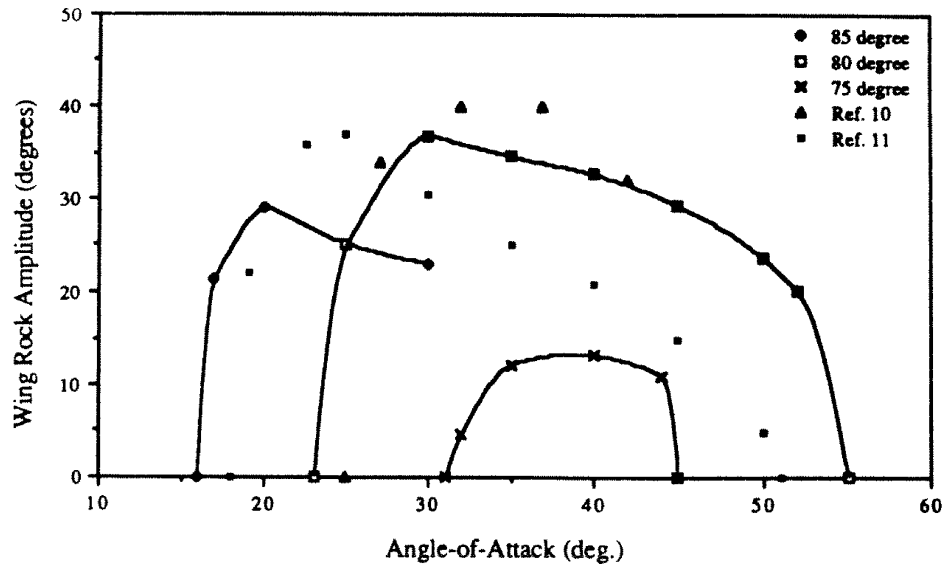


Fig. 37 Wing-rock amplitude as a function of angle of attack for wings with different sweep angles.<sup>120</sup>

wing rock and the proximity of leading-edge vortices. In fact, there may be a link to the hypothesized crossflow instability shown in Fig. 17. This instability was suggested as a possible mechanism for the quasi-periodic, antisymmetric motion of breakdown locations, which becomes larger and more coherent as angle of attack or sweep angle is increased.

Several aerodynamic phenomena have been suggested as the mechanism(s) leading to the self-induced oscillations, including vortex liftoff and vortex breakdown. Ng et al.<sup>120</sup> suggested that wing rock is initiated by vortex interaction, which results in asymmetries in vortex strength. Also, the importance of the proximity of the leading-edge vortices was noted. Interactions of forebody and wing vortices can also cause wing rock of slender configurations.<sup>121</sup> It has generally been agreed that vortex breakdown is not a necessary condition for wing rock. For slender delta wings, no breakdown was seen on the wing during the oscillations at the onset of the motion. Ericsson<sup>122</sup> suggested that the wing rock starts at the angle of attack at which the roll damping of the wing is lost. The mechanism(s) that start wing rock are likely to be different than those that sustain it. Arena and Nelson<sup>119</sup> suggested that a possible mechanism to sustain the wing-rock motion is the time lag in the position of the vortices normal to the wing surface.

## X. Unsteady Vortex Control

Here, a brief review of vortex control methods is presented, with emphasis on unsteady aspects. Use of unsteady effects for vortex control over stationary and maneuvering wings is considered. Efforts in this direction have two main goals: 1) control of leading-edge vortices to generate forces/moments for flight control and 2) control (delay) of vortex breakdown with stability and buffeting considerations. A review of research into vortex breakdown control is given by Mitchell and Delery.<sup>123</sup> Note that some flow control techniques may cause premature vortex breakdown at moderate incidences, whereas they may increase the vortex lift at low angles of attack due to increased strength of vortex.

Because the vorticity of the leading-edge vortices originates from the separation line along the leading edge, control of separation characteristics or shear layer can be used to influence the strength and location of the vortices as well as the location of vortex breakdown. Steady blowing and suction at the leading edge<sup>124–126</sup> are known to be effective tools. In addition, Gu et al.<sup>124</sup> applied periodic suction blowing in the tangential direction along the leading edge of the wing and reported a delay of vortex breakdown. The most effective period of the alternate suction blowing corresponded to  $fc/U_\infty = 1.3$ . It was shown that oscillatory blowing at the leading edge can enhance the lift at high angles of attack,<sup>127</sup> and optimum reduced frequency varied in the range of  $fc/U_\infty = 1–2$ .

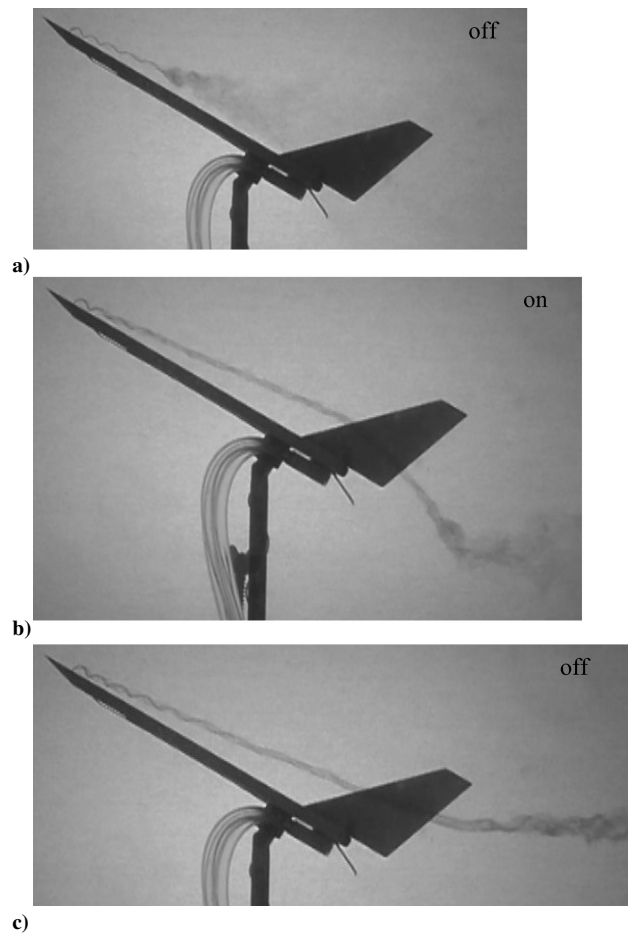


Fig. 38 Flow visualization of leading-edge vortex for a) jet off, b) jet on, and c) just after the jet is turned off.<sup>135</sup>

The relationship between these reported values of optimum frequency and characteristic frequencies of unsteady flow phenomena needs to be investigated. Leading-edge flaps are particularly attractive tools that can be used to influence the strength and location of these vortices. By using particle streak photography, Spedding et al.<sup>128</sup> reported that leading-edge vortices doubled in strength for a delta wing with flap. Deng and Gursul<sup>94</sup> showed that the circulation of the vortices became maximum at an optimum flap angle. Spedding et al.<sup>128</sup> considered large-amplitude harmonic oscillations

of flaps and revealed a 35% increase in vortex circulation over the equivalent stationary flap configuration. For small-amplitude flap oscillations, the strength of the vortices was larger than that of the quasi-steady case.<sup>94</sup> The effect of oscillating leading-edge flaps on breakdown was investigated for a delta wing with upward-deflected flaps.<sup>94</sup> The time-averaged breakdown location over one cycle may move upstream or downstream compared to the quasi-steady case, depending on the amplitude of flap oscillations and angle of attack. Combined, simultaneous use of leading-edge flaps and intermittent trailing-edge blowing to achieve optimum conditions for vortex control has been demonstrated by Vorobieff and Rockwell.<sup>129</sup> Variable leading-edge extension that effectively varies the sweep angle during a maneuver has been used to control leading-edge vortices and breakdown over a pitching delta wing.<sup>95</sup> Oscillations of sweep angle with the same frequency as pitching were employed, but with a phase angle. In a related study, by using the same delta wing model,

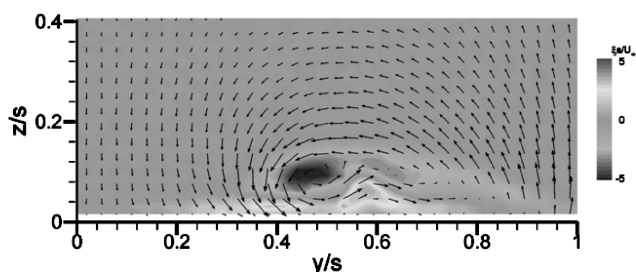
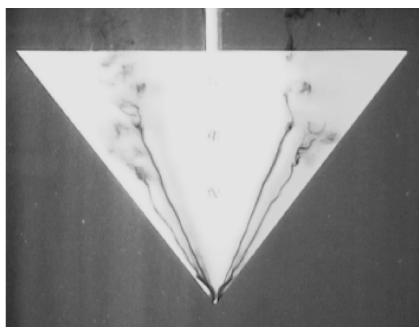


Fig. 39 Dual vortex structure (of the same sign of vorticity) over a nonslender delta wing<sup>136</sup> with a sweep angle of  $\Lambda = 50$  deg.

Srinivas et al.<sup>40</sup> demonstrated that active control of vortex breakdown can be achieved. The pressure fluctuations induced by the helical mode instability of vortex breakdown can be measured and used as a feedback signal for active control.

Flow control techniques applied near the wing apex received considerable attention. Because most of the vorticity within the leading-edge vortex originates from a small region near the apex of the wing, control of flow separation or shear layer can be successfully used without the need to apply flow control along the entire leading edge. Klute et al.<sup>130</sup> showed that a drooping apex flap can delay vortex breakdown and is equally effective in dynamic maneuvers.

Remaining vortex control techniques to be discussed are applied in locations/regions other than the leading edge. The earliest example is given by Werle,<sup>131</sup> who demonstrated a delay of vortex breakdown by applying suction along the vortex axis at a location downstream of the onset of vortex breakdown. Parmenter and Rockwell<sup>97</sup> conducted similar experiments and described the transient response of vortices to suction. Blowing from surface ports located beneath the vortex core at different angles<sup>132</sup> proved useful in delaying vortex breakdown. This technique was also effective for pulsed blowing during pitch-up maneuver.<sup>133</sup> Helin and Watry<sup>134</sup> and Shih and Ding<sup>11</sup> demonstrated that a trailing-edge jet can significantly delay vortex breakdown on a delta wing. Phillips et al.<sup>135</sup> showed that vortex breakdown can be delayed even in the presence of a fin, resulting in elimination or significant delay of fin buffeting. Figure 38 shows flow visualization images for jet off, jet on (in a steady-state case), and just after the jet is turned off. It is seen that the leading-edge vortex is drawn toward and parallel to the jet in the steady-state case. When the jet is turned off, the wing vortex realigns itself to become nearly parallel to the freestream. Vortex breakdown then slowly propagates upstream and eventually reaches a steady-state location similar to that shown in Fig. 38a. These results also illustrate the hysteresis and large phase lags associated with the wing vortical flow. The presence of large time constants associated with this complex jet-vortex interaction is important for the dynamic aspects of thrust vectoring, flight dynamics, and control.

## XI. Issues and Challenges

Much of our knowledge on vortex flows is related to slender vortices. Very little is known about the structure of vortices over non-slender delta wings ( $\Lambda \leq 55$  deg) and unsteady flow phenomena. Figure 39 shows an example of flow visualization for a  $\Lambda = 50$  deg delta wing, where a dual vortex structure is identified. Particle image velocimetry measurements<sup>136</sup> and direct numerical simulations<sup>137</sup> confirmed that both vortices have the same sign of vorticity. Another

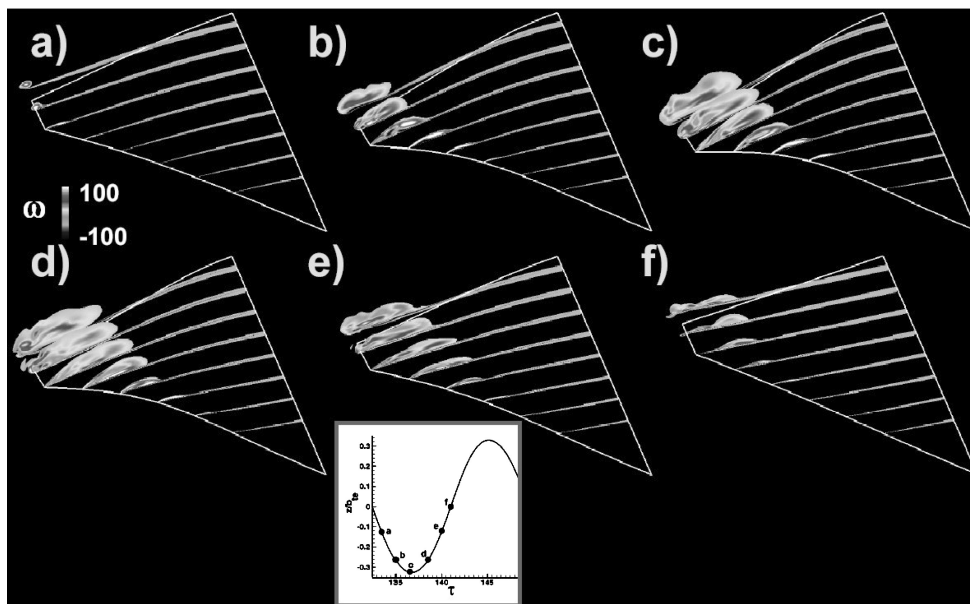


Fig. 40 Interaction of leading-edge vortex with a flexible delta wing.<sup>138</sup>

area that received little attention is the unsteady interactions for multiple vortices, such as found on double delta wings. Future unmanned aircraft will be highly maneuverable and highly flexible, with the capability of performing extreme maneuvers at high rates. At such high reduced frequencies, there is a possibility of coupling of aerodynamic maneuvers with the vortex instabilities reviewed in this paper. For flexible delta wings, vortex/wing interaction (Fig. 40) may lead to limit-cycle oscillations, where the vortex acts like an aerodynamic spring.<sup>138</sup> Figure 40 shows the growth and decay cycle of the leading-edge vortex over a flexible wing during the limit-cycle oscillations. Unsteady flow phenomena may interact and couple with structural vibrations. Finally, vortices at low Reynolds numbers are very relevant to micro air vehicles. Certain vortex flows show great sensitivity to Reynolds number, and unsteady aspects of these flows, as well as unsteady aerodynamics due to gust loads, are important.<sup>139</sup>

## XII. Conclusions

A wide range of unsteady phenomena relevant to slender vortex flows over stationary and maneuvering delta wings is reviewed. The origin, characteristics, and physical mechanisms of these unsteady phenomena and their role in buffeting are discussed. Dynamic response of leading-edge vortices for maneuvering wings and mechanisms of hysteresis and time lag effects are reviewed. Issues and challenges for unsteady vortex flows over delta wings are outlined.

Although the main features of the unsteady phenomena related to slender delta wings are relatively well understood, many aspects require further study. The details of the shear layer instabilities, steady and unsteady substructures, and transition need to be investigated further. Vortex wandering remains a problem in the characterization of the leading-edge vortices. Time-averaged velocity in the core of a wandering vortex is not representative of instantaneous core structure. The relationship between the shear layer instabilities and vortex wandering requires further study. The main instability associated with vortex breakdown is the helical mode instability, and the spiral form of breakdown is a consequence of this instability of the breakdown wake. The mechanisms of vortex interactions, which appear in the form of antisymmetric oscillations of breakdown locations, need to be investigated further. Whether vortex breakdown is a necessary part of these interactions is not known. Although these kinds of vortex interactions are thought to be dominant over slender wings, they also, surprisingly, exist over nonslender wings. These interactions may take very complicated forms when excited by external disturbances. The frequency spectrum of the unsteady flow phenomena that exist over stationary wings is very wide, which is one of the challenges in numerical simulations of these flows. Vortex breakdown, vortex interactions, and vortex shedding, either alone or in combination, play an important role in wing and fin buffeting, although vortex breakdown is the main source of buffeting over slender wings.

Vortex breakdown is the main source of hysteresis and time lag in unsteady flows for a maneuvering aircraft. Physical aspects of the time lag and hysteresis are not well understood, despite the fact that there appears to exist a universal mechanism in a variety of flows involving vortex breakdown. The main focus of this review paper has been the unsteady vortex flows over slender delta wings. There is very little known about the vortical flows over nonslender wings, let alone their unsteady aspects. Initial investigations suggest that significant differences in shear layer structure and vortex breakdown may exist. With the requirement for high maneuverability and increasing wing flexibility for future unmanned air vehicles, unsteady vortex flows and vortex/structure interactions will become more predominant.

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